

ESSAYS ON INDUSTRIAL ORGANIZATION
AND INFORMATION ECONOMICS

By

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To My Husband, Hsuan-Chi Chen, and My Parents

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CHAPTER 1 INTRODUCTION

Information asymmetry between suppliers and demanders often is severe and thus is an important issue that needs to be discussed. This dissertation is composed of two essays dealing with price competition in oligopoly when suppliers are better informed.

First, it is important to investigate the effect of competition among health maintenance organizations (HMOs), especially the competition for already enrolled customers, because HMOs are the most common form of managed-care companies and the managed-care industry is one of America's newest, largest industries in the 1990s. A primary concern is how the competition affects HMOs' incentives to invest in preventive care, which is often claimed as an important characteristic of HMOs. It takes time for an HMO to realize the benefit of investments in preventive care, while competition may easily break up the relationship between an HMO and its customers. When a customer switches from his original HMO to a new HMO due to an even lower price, the new HMO gets the benefit of investments of the original HMO. Thus, a holdup problem faces the original HMO, causing an underinvestment in preventive care in the short run and a potential rise in premiums later.

Another concern is about the number of HMOs under free-entry as compared to the social optimum. Since the US government embraces policies that move health care toward a market-based system, the roles played by the regulators are especially important in maintaining sound and smooth operations of the health care market. However, there is a question of whether they should increase or decrease the amount of regulation on

HMOs. If more HMOs will enter into the free entry market than is socially desired, a more stringent regulation should be set as a higher entry barrier. Alternatively, if fewer HMOs enter into the free entry market than is socially desired, policies of encouraging entrants should be proper.

The purpose of Chapter 2 is to develop a model to investigate the effects of competition among HMOs on their premiums and investments in preventive care, and to examine whether the number of HMOs under free-entry is socially optimal, considering the cases where HMOs can and cannot commit to the investment levels announced in their contracts.

Secondly, consumers often cannot determine with certainty before making their purchases the difference in qualities of products sold by different firms or at different stores. For instance, a consumer may read newspapers or consumer reports analyzing differentiated products a long time before purchase. When he actually shops and purchases the good, he may only remember that one good is of high quality and one is of low quality, instead of each product's exact quality. Alternatively, many products can be imitated by "pirates" in such a way that consumers cannot tell stores selling the fake from stores selling the authentic product. In these situations, consumers have to form beliefs about which product to buy without having complete information on product quality.

The question arises: can prices alone serve as a signal of product qualities in duopoly for consumers? If so, consumers can have confidence about their purchases. If not, is there any instrument that, together with price, can signal product quality? Furthermore, is the signal costly to the high-quality firm? Finally, are consumers better off under incomplete information than under complete information?

The purpose of Chapter 3 is to study the informational role of prices and money-back guarantees (or refunds) in the framework of duopoly with quality uncertainty for risk-neutral consumers. Specifically, I study two cases. The first case, inspired by Shieh (1996), considers homogeneous consumers with the same valuation for a working product, and two vertically differentiated firms with different unit costs of production, each of which simultaneously decides a refund policy followed by a price. The second case, inspired by Hertzendorf and Overgaard (1998a, 1998b), considers heterogeneous consumers with different valuation for a working product, and two vertically differentiated firms with the same unit costs of production, each of which simultaneously decides a refund policy and a price. The questions stated above are analyzed for each case. As one will see later, the results of the two cases are not all the same.

Finally, the dissertation concludes with a summary in Chapter 4.

CHAPTER 2

HMO COMPETITION, THE HOLDUP PROBLEM, AND FREE ENTRY

2.1. Introduction

Health insurance often is among the important benefits for employees. In 1980, about 80 percent of private insurance premiums were purchased through employers.¹ The most common level of health insurance was fee-for-service insurance, which up until 1990 covered over 90 percent of Americans whose jobs provided health coverage. This tendency, however, changed in the 1990s with the soaring of medical costs as premiums rose at 13.6 percent a year between 1988 and 1992. Managed-care companies induced a rapid switch of employers by offering relatively lower prices. By 1996, three quarters of workers with insurance were in health maintenance organizations (HMOs) or some other managed-care plan. Of the remaining quarter, most were in schemes that contain elements of managed-care. In 1998, only 3 percent of insured American remain in “unmanaged” fee-for-service plans. In other words, managed-care companies cover about 160 million people.²

Several factors contribute to the lower prices charged by HMOs (the most common form of managed-care companies). First, HMOs have more bargaining power in squeezing discounts from such suppliers as hospitals, pharmaceutical firms, and doctors. Second, HMOs have cut the number of unnecessary tests, operations, and days

¹ See Pauly (1986).

² See “Special” (03/07/1998).

of staying at hospitals. Third, HMOs focus more on preventive care, which may save medical costs in the long run. Finally, HMOs make better use of information technology to find the most cost-effective treatments.³

Since the managed-care industry is one of America's newest, largest industries in the 1990s, it is important to investigate the effect of competition among HMOs, especially the competition for already enrolled customers.⁴ A primary concern is how the competition affects HMOs' incentives to invest in preventive care, an important characteristic of HMOs as stated above. It takes time for an HMO to realize the benefit of investments in preventive care, while competition may easily break up the relationship between an HMO and its customers. When a customer switches from his original HMO to a new HMO due to an even lower price, the new HMO gets the benefit of investments of the original HMO. Thus, a holdup problem faces the original HMO, causing an underinvestment in preventive care in the short run and a potential rise in premiums later.

Another concern is about the number of HMOs under free-entry as compared to the social optimum. Since the US government embraces policies that move health care toward a market-based system, the roles played by the Federal Trade Commission (FTC) and the Department of Justice (DOJ) are especially important in maintaining sound and smooth operations of the health care market.⁵ However, there is a question of whether they should increase or decrease the amount of regulation on HMOs. If more HMOs will

³ See "Special" (03/07/1998) for more detail of the factors.

⁴ See "Special" (03/07/1998) for the report that there are about 1000 managed-care organizations in 1998.

⁵ See Gaynor and Haas-Wilson (1999) for roles these two institutions play.

enter into the free entry market than is socially desired, a more stringent regulation should be set as a higher entry barrier. Alternatively, if fewer HMOs enter into the free entry market than is socially desired, policies of encouraging entrants should be proper.

This essay develops a model to investigate the effects of competition among HMOs on their premiums and investments in preventive care, and to examine whether the number of HMOs under free-entry is socially optimal, considering the cases where HMOs can and cannot commit to the investment levels announced in their contracts.

A game-theoretic model is used to characterize the current situation. First, most customers enroll in HMOs through their employers, as stated above, who have no better information about the actual illness level of their employees than HMOs at the time of enrollment. After enrollment, customers are provided with preventive care, which gives the chosen HMO an opportunity to learn the exact illness level of each customer. Since the contract for health care coverage is often short in duration, the chosen HMO has to offer a new contract to employers at the end of the old contract. If an employer is not satisfied with conditions in the new contract (especially the premiums), he will switch his employees to another HMO.⁶

Section 2.2. presents the formal model of competition among health plans offered by HMOs. In particular, for simplicity, I assume that every customer, who is a small employer with few employees, chooses only one plan,⁷ and every HMO, which is a

⁶ This essay only tries to model the most general situation. Thus, we ignore such specific situations as employees quitting their jobs.

⁷ "Special" (03/07/1998) reports that roughly half of the firms in the US offer their employees only one health care plan and another 20 percent of the firms offer a choice of two plans, according to KPMG, a consultant. This essay only considers the former situation.

combination of health care insurance and affiliated hospitals,⁸ offers only one plan.

There are three stages in the model. In stage 1, n symmetric HMOs enter simultaneously into the health care market. In the beginning of stage 2, each HMO announces a health plan for preventive care services. Each plan can be characterized by the horizontal characteristic of its services, the preventive care price for the plan, and investments in preventive care under the plan.⁹ Two investments are considered: general investment I , applied to the whole customer population, which decreases the chance of any customer getting sick; and customer-specific investment e , applied only to the customer himself, which decreases the cost of future treatment. Each customer is characterized by his illness level and his preferences for the horizontal characteristics of health plans.¹⁰ When signing the contract for a plan, both parties know only the distribution of illness, not the exact illness level of the customer. This information is learned by the original HMO of the customer after its investments are made, and will be known to the customer or other HMOs only after the customer leaves the original HMO and reveals it with a considerable cost. A customer's decision of choosing his original HMO will also be affected by an HMO's credibility of commitment to the levels of the two investments stated in the plan.

In stage 3, knowing the illness level of each customer, the chosen HMO offers a take-it-or-leave-it premium to the customer who still has no information about his own

⁸ This means a group- or staff-model HMO.

⁹ This essay does not consider vertical differentiation. That is, quality is assumed the same across all plans. This can be justified by the attitude of most employers, who generally care more about price than about quality.

¹⁰ Since every customer is an employer, the illness type and preferences of a customer represent the average cost of illnesses and average tastes of his employees.

illness level. A customer who rejects the offer can verify his level of illness by incurring a cost, and can switch to another HMO with a bargained price.

Section 2.3. characterizes the equilibrium strategies of the chosen HMO and the customer. Following the argument in Reinganum and Wilde (1986), I find a unique separating equilibrium in which HMOs offer each customer a premium that is linear and increasing with his exact illness level, while each customer, without knowing his own illness level, accepts the offer with a probability that is nonincreasing with the premium at a decreasing rate. It is noteworthy that, in equilibrium, a customer is more likely to accept the offer from his original HMO when he is healthier, which results in a higher expected profit for the HMO. This is because a customer without information tends to accept an offer with a low premium. It implies that an HMO has an incentive to dump severely ill patients if it would have known the information before signing the contract. Moreover, the stronger bargaining power of the customer (or, the more HMOs in the market), the lower the offer price and the less likely that the customer will accept the offer.

Section 2.4. analyzes the optimal decisions of HMOs in stage 2, including the optimal preventive care price and the optimal investments. There are two cases with regard to the decisions, depending on whether HMOs can commit to the investment levels announced in the contract. It is found that, generally, the optimal investments increase with the verification cost and with a customer's utility of being healthy but decrease with transferability of preventive care and with the number of HMOs in the market. In the commitment case, however, because HMOs take into account the business-stealing effect, in which customers value investments when choosing an HMO,

the optimal investments may increase with transferability of preventive care and with the number of HMOs in the market. When comparing the optimal investments in these two cases with the social optimum, it is clear that, first, HMOs underinvest in the noncommitment case. Second, investment levels are higher in the commitment case than in the noncommitment case, due to the business-stealing effect. More interestingly, if the business-stealing effect is too strong, HMOs can overinvest in the general investments, but not the customer-specific investments.

Section 2.5. analyzes the number of HMOs when firms are free to enter and exit the market, as compared to the social optimum when the regulator limits the number of HMOs so as to maximize social welfare. I find that, under certain conditions, HMOs can be profitable in the social optimum which induces too many HMOs to enter when there is free entry. Moreover, profits of an HMO can decrease with competition, under certain conditions. Last, consolidations among HMOs may not increase premiums. Section 2.6. summarizes the findings and outlines future research direction. Formal proofs of the findings are provided in Appendix A.

This essay is related to several different literatures. In methodology, for the optimal offer price, I apply the signaling equilibrium with the refinement of universal divinity, which is developed by Banks and Sobel (1987) and, for example, used in Banks (1992) and Reinganum and Wilde (1986).¹¹ Moreover, the location model is used in characterizing the horizontal characteristic of a health plan, as employed by Che and Gale

¹¹ The Intuitive Criterion developed by Cho and Kreps (1987) can also be used to refine signaling equilibria, but it is not as strong as the concept of the universal divinity in selecting the outcome with the least amount of inefficient signaling. See Fudenberg and Tirole (1991) for more detail.

(1997).¹² Finally, the argument used in comparing the number of HMOs in the free-entry equilibrium and the social optimum is similar to Mankiw and Whinston (1986).

Before proceeding, I contrast my analysis with two related studies of HMO competition. First, Encinosa and Sappington (1997) develop a model of competition between two HMOs to analyze the effects of market power, scale economies, and asymmetric information of health risk on market outcomes. In their setting, there are two levels of health-risk patients who decide which health-care plan to purchase from two HMOs. They assume that patients have better information about their own health risk than HMOs, while my assumption is the opposite, that employers choose health care plans.

Second, Che and Gale (1997) study how the employer, viewed as a sponsor of a buyer alliance, manages competition among insurance firms. They find a tendency toward too many firms, too much quality, and a bias toward a format involving the prescreening of insurance plans by the employer. They focus on one big employer whose consumers have the same illness level, while I focus on many small employers with different illness levels as a contrast. Moreover, they do not consider investments of preventive care as I do, which is often claimed to be one of the most important characteristics of HMOs.

¹² Ma and Burgess (1993) and Wolinsky (1997) also use location model to examine a spatial duopoly with a vertical quality dimension. But, their firms locate at the end of an interval, instead of around a circle.

2.2. The Model

I consider a model of competition among health plans offered by HMOs. The players in the model are customers and HMOs. Each customer can be thought of as an employer who provides health plans for its employees as benefits, or as a sponsor who provides health plans for its members. Each HMO is affiliated with hospitals or clinics, which provide health services.¹³ For simplicity, I assume that every customer, who is a small employer with few employees, chooses only one health plan and every HMO offers only one health plan.

Initially, in stage 1, n HMOs simultaneously enter the health care market with an entry cost J for each firm. While n is exogenously fixed for now, I will discuss the equilibrium and the optimal determination of n in section 2.5.

Then, in the beginning of stage 2, each HMO announces a health plan for preventive care services, such as regular checkups, immunizations, and medical education, provided by affiliated hospitals under that plan. Each health plan can be characterized by the characteristic of its services, the preventive care price that is charged for the plan, and investments in preventive care under the plan.

First, the characteristic of a plan captures a horizontal element of the services, such as geographic locations of the affiliated hospitals, their practice styles, or their specialties. It is modeled as a location $h_i \in [0, 1]$ (measured clockwise) on a unit circle,

¹³ As in Che and Gale (1997), since my focus is on competition among HMOs, I abstract from the possible agency problem that exists between an HMO and its member hospitals or clinics.

where $i \in \{0, 1, 2, \dots, n-1\}$.¹⁴ To simplify the analysis, I ignore the characteristic of a plan capturing the vertical element of the services, such as quality, by assuming it is the same for all plans. I also focus on a symmetric equilibrium in the sense that firms locate equidistantly, charge the same price to each customer, and make the same number of investments on behalf of each of their customers.¹⁵ Hence, the distance between any two adjacent HMOs is $1/n$ and, for convenience, the location of HMO_{*i*} is denoted as $h_i = i/n$, where $i \in \{0, 1, 2, \dots, n-1\}$.

Second, the preventive care price $P_i \geq 0$ represents an up-front payment for the plan. As mentioned earlier, I focus on a symmetric equilibrium in which each HMO charges the same preventive care price to each of its customers. This consideration can be justified in the case where discrimination is prohibited.

Last, the two investments in preventive care under the plan are the general investment I and the customer-specific investment e , which will be made after the plan is purchased by a customer, but before the exact illness level of the customer is learned. Rather, these investments facilitate an HMO to learn it privately. The features of these two investments will be described later. In sum, plan i is characterized by a quadruple (h_i, P_i, I_i, e_i) , with $P_i = P$, $I_i = I$ and $e_i = e$ in the symmetric equilibrium.

¹⁴ Note that the terms "location" and "circle" can be interpreted in many different ways depending on the product space of interest.

¹⁵ As in Salop (1979), the point of this model is not to look at the particular location choice, but to study the extent of the holdup problem and entry. Omitting the choice of location allows us to study these two issues in a simple way. Economides (1989) justifies that equidistant location is indeed an equilibrium in a circular model with quadratic transportation costs.

Customers are the potential buyers for those health plans. Each customer is characterized by his preferences and his illness level. Note that since every customer is an employer, the preferences and the illness level of a customer are represented by the average tastes and the average cost of illnesses of his employees. Customers' preferences are described by their different tastes for the horizontal characteristics of health plans. For instance, suppose health plans are distinguished by geographic locations; then a customer may prefer a plan under which the affiliated hospitals are close to his home. Taking another example. Suppose health plans are distinguished by their specialties in health services; then customers may differ in their tastes for those different specialties. In accordance with the horizontal characteristics of health plans, I model preferences of each customer by his ideal location on the unit circle, $x \in [0, 1]$, also measured clockwise. I assume that customers' ideal locations are uniformly distributed, with density 1. When a customer located at x goes to HMO_i for preventive care services, he incurs a once-and-for-all disutility, $t(x, i)$, for not being able to get his ideal plan, which is defined as the distance between him and HMO_i . That is,¹⁶

$$t(x, i) \equiv \min\{|x - i/n|, 1 - |x - i/n|\} \quad i \in \{0, 1, 2, \dots, n-1\}.$$

Finally, every customer has a cost of illness c , which is represented by a cumulative probability distribution $F(c)$ with density $f(c) > 0$ on the support $[\underline{c}, \bar{c}]$ (with $F(\underline{c})=0$ and $F(\bar{c})=1$). I assume the distribution is independent of the customer's location x .

¹⁶ In the literature, there are two common types of transportation costs, namely, the linear form and the quadratic form. To simplify the analysis, this essay adopts the linear form as in Salop (1979), and assumes a covered market to avoid the problem of a discontinuous demand function.

After health plans are announced, each customer chooses an HMO for health services to maximize his monetary utility. At the time of contracting, the locations of both parties of the plan and the distribution of illness level of the customer are common knowledge to both parties. However, no party knows the exact illness level of the customer at that time. This information is learned by the original HMO of the customer after its investments are made, and is known to the customer or other HMOs only after the customer leaves the original HMO and verifies it at a considerable cost.

A customer's decision in choosing his original HMO is affected by three factors. The first factor, of course, is the preventive care price. The higher the preventive care price, the less likely a customer will accept the contract. The second factor is the disutility incurred due to not being able to purchase his ideal plan. The higher disutility makes it less likely for the customer to purchase the plan. The last factor is an HMO's credibility of commitment to the levels of the two investments stated in the plan. Customers value the actual investments made by HMOs, which may be different from announced investments if the commitment is incredible. Thus, the stage-2 utility, U_2 , for a customer located at x in considering plan i characterized by (h_i, P_i, I_i, e_i) is

$$U_2 = \begin{cases} S - P_i - t(x, i) + EU_3(I^*, e^*) & \text{if HMOs cannot commit to investments} \\ S - P_i - t(x, i) + EU_3(I_i, e_i) & \text{if HMOs can commit to investments} \end{cases},$$

where S is a sufficiently large constant so that each customer will purchase a plan, $EU_3(\cdot, \cdot)$ is the ex ante expected stage-3 utility and will be described later, and I^* and e^* are the optimal actual investments decided after contracting. Since this essay only considers the symmetric equilibrium, I can omit firm-specific subscript for the optimal actual investments. Furthermore, I assume that the discount factor is 1, which is also common knowledge.

The features of the two investments made under each plan in stage 2 are described as follows. First, the general investment I , such as medical education, applied to the whole customer population, reduces chances of being severely ill. That is, I presume first-order stochastic dominance: $F_I(c, I) > 0$ for $c \in (\underline{c}, \bar{c})$. I also assume decreasing return to investment, i.e., $F_{II}(c, I) < 0$.¹⁷

Second, the customer-specific investment e is used to decrease the cost of treating the customer later on. The cost of treatment for a customer in stage 3 depends on whether he stays with his original HMO or switches to another HMO. That is, the cost of treatment is

$$\text{Treatment cost} = \begin{cases} c - \phi(e) & \text{if he stays with his original HMO} \\ c - \mu\phi(e) & \text{if he switches to another HMO,} \end{cases}$$

where $0 < \mu < 1$, $\phi(0) = 0$, $\phi'(\cdot) > 0$, $\phi''(\cdot) < 0$.¹⁸ This means that a customer with the cost of illness c can reduce his cost of treatment by $\phi(e)$ if he stays with his original HMO, where the size of reduction in the cost of treatment depends on the amount of the customer-specific investment e . Like the effect of the general investment on the probability distribution of illness, the reduction in the cost of treatment, $\phi(e)$, increases with the customer-specific investment e at a decreasing rate. If the customer switches to another HMO in the next stage, however, the size of the reduction in the cost of treatment will be smaller than it otherwise would be if he stays in the original HMO. For example, the new

¹⁷ These two assumptions can be seen in Laffont and Tirole (1993, p. 86), and Grossman and Hart (1983).

¹⁸ I assume that the treatment cost is always nonnegative, i.e., $c - \phi(e^*) \geq 0$ for all c and the optimal e . This means that the cost saving never exceed the cost itself no matter how large the customer-specific investment is. Particularly, it implies that $\underline{c} - \phi(e^*) \geq 0$.

doctor may need time to get acquainted with the patient's records or may require different tests to reconfirm the level of illness. This smaller reduction reflects the transition costs for a customer that a new HMO will have to incur. Thus, I call μ *the transferability of preventive care*.

Note that investments e and I are not specific to the illness level c of any customer as an HMO makes investments before it learns the illness level. Thus, e and I are not functions of c .

In stage 3, after an HMO privately observes the illness level of each of its own customers, it offers a contract extension for future treatment with each of its own customers by proposing a take-it-or-leave-it offer. More specifically, taking an example of HMO_0 and Alan, who is one of HMO_0 's customers, HMO_0 offers a price O to Alan after it observes Alan's illness level c . Then, according to this offer price O , Alan decides a probability, α , of accepting the offer. To reduce the complexity of the model, I assume that investments are observable to both parties of the plan, but unobservable to other HMOs or customers.

Suppose each customer's monetary utility for getting healthy is H . Then, in the above example, Alan's utility for accepting the offer from HMO_0 will be $(H-O)$, and HMO_0 gets $[O-(c-\phi(e))]$ as a profit. Note that this profit should be nonnegative for all $c \in [\underline{c}, \bar{c}]$ as no HMO makes an offer to get negative profits.¹⁹ If Alan rejects the offer, he will first pay a cost V to verify his illness level and the investment e and I made by HMO_0 . Then, using this information, along with the information of his location if necessary, he bargains with the rest of $(n-1)$ HMOs and then trades with one of them at a

¹⁹ I assume that there is no regulatory policy to intervene between HMOs and customers.

price set according to the Nash bargaining solution. Suppose that the customer shares $\lambda(n)$ proportion of the gains from trade, which is $[H-V-(c-\mu\phi(e))]$, where $\lambda(n) \in (0,1)$ and $\lambda'(n) > 0$.²⁰ Hence, in the above example, Alan's utility for rejecting the offer from HMO₀ will be $\lambda(n)[H-V-(c-\mu\phi(e))]$, and the other HMO to which Alan switches gets $(1-\lambda(n))[H-V-(c-\mu\phi(e))]$ as a profit. That is, the bargaining price is $\{H-V-\lambda(n)[H-V-(c-\mu\phi(e))]\}$. For technical reasons, I assume $0 < \frac{1-\lambda}{\lambda} \frac{H-c+\phi(e)}{G} \leq 1$, which implies a stronger bargaining power on the customer side, i.e., $\lambda(n) \in (1/2, 1)$. This assumption is essential in comparing the optimal preventive care price in the noncommitment case, P_{nc}^* , to that in the commitment case, P_c^* . Moreover, it is essential in proving the decrease of the overall expected profit of an HMO with the number of HMOs, and in comparing the number of HMOs in the free entry equilibrium to the social optimum. That a small employer has a stronger bargaining power than his switched HMO can be justified by the fact that the employer has outside opportunities and owns extra benefits, $\mu\phi(e)$, to share with the switched HMO. From the above, then, the ex ante expected stage-3 utility, $EU_3(I_b, e)$, for a customer is

$$EU_3(I, e) = \int_{\bar{e}}^{\bar{e}} [a(H - O) + (1 - a)\lambda(H - V - c + \mu\phi(e))] dF(c, I).$$

Note that the disutility of not being able to get the ideal plan is not considered in the expected stage-3 utility. As explained earlier, the disutility is incurred once and for

²⁰ I assume that H is sufficiently large so that this surplus is positive. Moreover, the assumption of $\lambda'(n) > 0$ is to capture the idea that the more HMOs in the market, the stronger bargaining power for the customer. Therefore, this is not the standard Nash bargaining solution as suggested in Nash (1953) which splits surplus evenly. In fact, it is in the family of asymmetric Nash solutions studied by Harsanyi and Selten (1972). See, e.g., Osborne and Rubinstein (1990) for more detail.

all in stage 2 when a customer chooses his original HMO. For example, a customer with heart disease may choose one HMO whose doctors help control the cholesterol level through medicine, or another HMO whose doctors help controlling the cholesterol level through exercises. But, when it comes to the treatment stage, these HMOs are the same in technology. That is, the customer has to get the same operation for his heart disease no matter which HMO he is enrolled in. Thus, there are no such different disutilities associated with different HMOs in stage 3.

Figure 2.1. depicts the time line of the model.

2.3. Analysis Of The Optimal Offer Price

Backward induction is used to analyze the equilibrium outcome, which includes the optimal offer price, the optimal investments, and the optimal preventive care price. This section characterizes the optimal offer price in the third stage constructed using the equilibrium notion for a signaling game. Due to symmetry and without loss of generality, I focus on the behaviors of an HMO and one of its customers with illness level c , i.e., HMO_0 and Alan in the above example.

When it is in the third stage, HMO_0 learns the illness level c of its customer through preventive care investments made in the second stage. This customer, however, does not have such information until he verifies it. Hence, this is a typical signaling game in that the informed party, HMO_0 , acts first, and then the uninformed party, the customer, responds. The only information asymmetric to parties is the level of illness c , not investments I and e . By construction of the model, location x does not matter in determining the optimal offer price, either.

In this signaling game, a strategy for HMO₀ is a function $O = O(c): [\underline{c}, \bar{c}] \rightarrow \mathcal{R}$, which specifies an offer price for each possible level of illness; and a strategy for a customer is a function $a = a(O): \mathcal{R} \rightarrow [0, 1]$, which specifies the probability that this customer accepts the offer O .²¹ Because the customer does not know the true illness level c , he must form some beliefs about c on the basis of the offer price O . Define point beliefs $c = b(O)$, which assign a unique illness level to each offer price. Given the beliefs $b(\cdot)$, the expected utility for the customer when the offer O is made and he accepts it with probability a , is

$$\Pi^c(O, a; b) = a\{H - O\} + (1 - a)\{\lambda(n)[H - V - (b(O) - \mu\phi(e))]\}. \quad (2.1)$$

Expected profit (without costs of entry and investments) per customer for HMO₀, which knows the illness level c , makes the offer price O , and takes the strategy $a(O)$ of the customer as given, is

$$\Pi^0(c, O; a) = a(O)[O - (c - \phi(e))]. \quad (2.2)$$

Following the argument in Reinganum and Wilde (1986), I restrict attention to separating equilibria, in which HMO₀'s offer price "signals" the customer's true illness level. Pure or partial pooling equilibria are excluded by appealing to universally divine equilibrium, the refinement of sequential equilibrium proposed by Banks and Sobel (1985). I give the argument for eliminating pooling equilibria in the proof of Proposition 2.1 in Appendix A. A triple (b^*, a^*, O^*) is a *separating equilibrium* if: (a) given the belief b^* , the probability of accepting policy $a^*(\cdot)$ maximizes the customer's expected

²¹ Note that although the strategy for HMO₀ is a function of illness level c , HMO₀ only presents one specific offer price O to each customer, instead of a menu. This also applies to the acceptance probability a .

utility; (b) given $a^*(.)$, the offer price policy $O^*(.)$ maximizes HMO₀'s expected profit from the customer; and (c) $b^*(O) \in [\underline{c}, \bar{c}]$ for all O with $b^*(O^*(c)) = c$ for all $c \in [\underline{c}, \bar{c}]$; that is, the customer must assign an existing illness level to every offer price O , and the beliefs must be correct for offer prices that are made in equilibrium.

To construct a candidate for equilibrium, I first consider the decision problem facing the customer. Differentiating the customer's expected utility Π^c with respect to his decision variable a gives

$$\partial \Pi^c / \partial a = H - O - \lambda(n)[H - V - (b(O) - \mu\phi(e))]. \quad (2.3)$$

This expression is independent of a ; if it is positive, then $a^*(O)=1$; if it is negative, then $a^*(O)=0$; if it is zero, then the customer is indifferent about the value of $a^*(O)$. Consider initially an interior solution, in which $a^*(O) \in (0, 1)$. Then $O^*(c)$ must satisfy $\partial \Pi^c / \partial a = 0$, which after incorporating the consistency condition that $b^*(O^*(c)) = c$, yields

$$O^*(c) = H - \lambda(n)[H - V - (c - \mu\phi(e))]. \quad (2.4)$$

However, the offer price function $O^*(c)$ must also maximize HMO₀'s expected profit, given $a^*(O)$. If $a^*(O)$ is differentiable, then $O^*(c)$ must also solve

$$\partial \Pi^0 / \partial O = a'(O) [O - c + \phi(e)] + a(O) = 0. \quad (2.5)$$

Combining equations (2.4) and (2.5) yields a first-order linear differential equation, which $a^*(O)$ must satisfy:

$$a'(O) [G] + a(O) = 0, \quad (2.6)$$

where $G(c, n, e, \mu, H, V) \equiv H - (c - \phi(e)) - \lambda(n)[H - V - (c - \mu\phi(e))]$. Notice that $G(\dots) = O^*(c) - (c - \phi(e)) \geq 0$ for $c \in [\underline{c}, \bar{c}]$ must hold, because it is an HMO's profit when its proposed offer is accepted by the specific customer. There is a unique solution to equation (2.6) through

any boundary point. It is a dominant strategy for the customer to select $a^*(O) = 1$ for all O satisfying

$$O \leq H - \lambda(n)[H - V - (\underline{c} - \mu\phi(e))]. \quad (2.7)$$

Since the term $\lambda(n)[H - V - (\underline{c} - \mu\phi(e))]$ in the right hand side of inequality (2.7) represents the highest possible utility that the customer will get if he rejects HMO₀'s offer. Define $\underline{O} = H - \lambda(n)[H - V - (\underline{c} - \mu\phi(e))]$. Hence, $a^*(\underline{O}) = 1$ in equilibrium, otherwise if $a^*(\underline{O}) < 1$, HMO₀ encountering with a customer of $\underline{c}$ would lower its offer slightly and insure the acceptance of the customer, thus resulting in higher profits.

Consequently, the solution to equation (2.6) with boundary condition $a^*(H - \lambda(n)[H - V - (\underline{c} - \mu\phi(e))]) = 1$ becomes:

$$a^*(O) = \exp \left\{ \frac{H - \lambda(n)[H - V - (\underline{c} - \mu\phi(e))] - O}{G} \right\}. \quad (2.8)$$

The optimal acceptance probability can also be expressed as an indirect function of illness level c :

$$a^*(O(c)) = \exp \left\{ \frac{\lambda(n)(\underline{c} - c)}{G} \right\}.$$

Furthermore, I need to specify the (out-of-equilibrium) beliefs about the offer prices outside the range $[\underline{O}, \bar{O}]$, where $\bar{O} = H - \lambda(n)[H - V - (\bar{c} - \mu\phi(e))]$. Among the permissible beliefs which satisfy the requirement of the sequential equilibrium concept (i.e., beliefs do not assign any probability to levels that are known not to exist), there is a set of beliefs simple and intuitive: if $O < \underline{O}$, let $b^*(O) = \underline{c}$, and if $O > \bar{O}$, let $b^*(O) = \bar{c}$. The idea of this set of beliefs is that, when a price is offered that should not be offered by an HMO facing any illness level of a customer in equilibrium, the customer believes

himself to be the level such that the HMO's equilibrium price is closest to the offered one.

It is readily verified that equations (2.4) and (2.8), together with out-of-equilibrium beliefs stated above, characterize the separating equilibrium. Moreover, this is the unique equilibrium, once the refinement of universal divinity is applied. The equilibrium is "unique" in the sense that it is one of a unique "equivalence class" of separating equilibria which have the same policies $O^*(.)$ and $a^*(.)$.²²

Proposition 2.1. *The following triple (O^*, a^*, b^*) is the "unique" separating equilibrium: define $\underline{Q} = H - \lambda(n)[H - V - (\underline{c} - \mu\phi(e))]$ and $\bar{O} = H - \lambda(n)[H - V - (\bar{c} - \mu\phi(e))]$. Then I have:*

- (1) $O^*(c) = H - \lambda(n)[H - V - (c - \mu\phi(e))]$ for $c \in [\underline{c}, \bar{c}]$;
- (2) $a^*(O) = 0$ for $O > \bar{O}$; $a^*(O) = \exp\left\{\frac{H - \lambda(n)[H - V - (\underline{c} - \mu\phi(e))] - O}{G}\right\}$ for $O \in [\underline{Q}, \bar{O}]$; and $a^*(O) = 1$ for $O < \underline{Q}$;
- (3) $b^*(O) = \bar{c}$ for $O \geq \bar{O}$; $b^*(O) = H - V + \mu\phi(e) - [(H - O)/\lambda(n)]$ for $O \in [\underline{Q}, \bar{O}]$; and $b^*(O) = \underline{c}$ for $O \leq \underline{Q}$.

Proofs of all results are provided in Appendix A. Figure 2.2. depicts the equilibrium strategies $O^*(c)$ and $a^*(O)$. It can be easily seen that the offer price O is a linear increasing function of illness level c with a slope less than 1. That is, the offer price is the lowest for the healthiest customer. Moreover, the offer price only increases by $\lambda(n) < 1$ for every one-dollar increase in cost of illness. It is interesting to note that the

²² See Reinganum and Wilde (1986) for more details.

acceptance probability a is a nonincreasing function of the offer price O at a decreasing rate, with a jump at the highest possible offer $\bar{O}$. That is, healthier customers are more likely to accept their original HMOs' offers. This is because HMO_0 offers a lower price to the healthier customer so that the customer, not knowing his own illness level, is more likely to accept its offer.

Compared to the bargaining price, $H-V-\lambda(n)[H-V-(c-\mu\phi(e))]$, the equilibrium offer price $O^*(c)$ is just higher than the bargaining price by V , the verification cost. In other words, in equilibrium, HMO_0 demands an offer price equal to the total cost (i.e., the bargaining price $H-V-\lambda(n)[H-V-(c-\mu\phi(e))]$ and the verification cost V) the customer would have to incur if he rejects HMO_0 's offer and switches to a new HMO. Another point of view for O^* is that, outside opportunities to a customer improve the customer's status quo point so as to limit an HMO's ability of charging a higher offer price, though an HMO has an advantage over the offer price by having the private information and initiating the negotiation. Precisely, the difference of the two offer prices an HMO can charge without and with outside opportunities is exactly $\lambda(n)[H-V-(c-\mu\phi(e))]$, an amount equal to what a customer can get from other HMO if switched. Apparently, this difference increases with a customer's bargaining power or the number of HMOs in the market. But, it decreases with illness level.

It is interesting to see changes of the offer price and the acceptance probability with respect to the customer's bargaining power, $\lambda(n)$, or the number of HMOs in the market, n . First, the differentiation of O^* with respect to $\lambda(n)$ is negative for $(\underline{c}, \bar{c})$, which implies that the stronger bargaining power of the customer (or, the more HMOs in

the market), the lower the offer price.²³ Second, the differentiation of a^* with respect to $\lambda(n)$ is negative for $(\underline{c}, \bar{c})$, which implies that the stronger bargaining power of the customer (or, the more HMOs in the market), the less likely for the customer to accept the offer from his original HMO.²⁴ These results are consistent with one's intuition.

While Proposition 2.1 and equation (2.1) indicate that the stage-3 expected utility for a customer with illness level c is

$$\Pi^c(O^*(c), a^*(.); b(.)) = \lambda(n)[H - V - (c - \mu\phi(e))], \quad (2.9)$$

Proposition 2.1 and equation (2.2) show that the expected profit (excluding the costs of entry and investments) for HMO₀ from a customer of level c , is

$$\Pi^0(c, O^*(.); a^*(.)) = \exp\left\{\frac{\lambda(n)(\underline{c} - c)}{G}\right\} G, \quad (2.10)$$

where $G(c, n, e, \mu, H, V) \equiv H - (c - \phi(e)) - \lambda(n)[H - V - (c - \mu\phi(e))]$. Equation (2.9) shows that, for every additional dollar in the cost of illness, a customer only loses his utility by $\lambda(n) < 1$, which means that he pegs his shares of risk in the cost of illness at $\lambda(n)$ with either his original HMO or the other HMO to which he switches. Ironically, the more bargaining power for a customer, the more the customer shares with his HMO in the cost of illness, though it is still less than 1.

Equation (2.10) indicates that an HMO's expected profit (excluding costs of entry and investments) decreases with a customer's illness level because both of the acceptance probability and the profit from an accepted offer are decreasing with the customer's

²³ More exactly, $\frac{\partial O^*}{\partial \lambda} = -[H - V - (c - \mu\phi(e))] < 0$ for $(\underline{c}, \bar{c})$.

²⁴ More specifically, $\frac{\partial a^*}{\partial \lambda} = \exp\left\{\frac{\lambda(\underline{c} - c)}{G}\right\} \frac{(c - \underline{c})[H - (c - \mu\phi(e))]}{G^2} < 0$ for $(\underline{c}, \bar{c})$, where $[H - (c - \mu\phi(e))] = [H - V - (c - \mu\phi(e))] + [V + (1 - \mu)\phi(e)] > 0$.

illness level. This implies that an HMO's expected profit (excluding costs of entry and investments) is maximized when the customer is the healthiest.²⁵ This especially gives an HMO an incentive to dump a less healthy customer before making investments if it would have known this private information.²⁶

2.4. Analysis Of The Optimal Decisions In Stage Two

Having analyzed the equilibrium strategies for a specific customer and an HMO at stage 3, I now turn to the analysis of the optimal decisions for an HMO at stage 2. These decisions include the optimal investments and the optimal preventive care price. Two cases are considered, according to the credibility of HMOs to commit to the investment levels announced in the beginning of stage 2. Comparisons of the optimal investments in these two cases with the social optimal investments are also provided at the end of this section.

2.4.1. The Noncommitment Case

It is possible that HMOs cannot commit to the levels of investments announced in the contract. For instance, HMOs may adjust their investment levels for an improvement in the technology of treatment, which occurs after the plan is purchased, but before the investments are made. In this case, HMOs will solve for their optimal decisions step by step, according to the timing of the decisions. Therefore, by backward induction, HMOs

²⁵ More precisely, $\partial \alpha^* / \partial \lambda = \lambda \exp\left(\frac{\lambda(\varepsilon - c)}{G}\right) \frac{-G + (1 - \lambda)(\varepsilon - c)}{G^2} < 0$, $G_c = -(1 - \lambda) < 0$, and $\partial \Pi^0 / \partial c = -\exp\left(\frac{\lambda(\varepsilon - c)}{G}\right) \frac{G - \lambda(1 - \lambda)(\varepsilon - c)}{G} < 0$.

²⁶ See Sappington and Lewis (1999) on patient antidumping.

first solve for the optimal investments, and then the optimal uniform preventive care price.

2.4.1.1. The optimal investments

Recall from equation (2.10) about the expected profit (excluding the costs of entry and investments) for an HMO from a customer of level c , then an HMO's problem is to select I and e to

$$\text{Maximize}_{e, I} \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(n)(\underline{c}-c)}{G}\right) G dF(c, I) - e - I, \quad (2.11)$$

where $G(c, n, e, \mu, H, V) \equiv H - (c - \phi(e)) - \lambda(n)[H - V - (c - \mu\phi(e))]$.

The optimal interior solutions for I and e , denoted as I_{nc}^* and e_{nc}^* , are characterized by the following first-order conditions

$$\lambda_I \equiv \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(n)(\underline{c}-c)}{G}\right) G dF_I(c, I) - 1 = 0,$$

or, by using integration by parts,

$$\lambda_I \equiv \int_{\underline{c}}^{\bar{c}} F_I(c, I) \left\{ \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) [1 - (1 - \lambda) \frac{\lambda(\underline{c}-c)}{G}] \right\} dc - 1 = 0, \quad (2.12)$$

$$\lambda_e \equiv \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(n)(\underline{c}-c)}{G}\right) (1 - \lambda\mu)\phi'(e) \left(1 - \frac{\lambda(\underline{c}-c)}{G}\right) dF(c, I) - 1 = 0. \quad (2.13)$$

From equations (2.12) and (2.13), it is hard to find closed-form solutions for the optimal general investment I_{nc}^* and the optimal customer-specific investment e_{nc}^* , unless a more specific structure of the model is introduced. Nonetheless, this does not affect the understanding of comparative statics, which is reported in the following proposition.

²⁷ In order to satisfy the second-order condition, i.e., $\lambda_{ee} < 0$, $\lambda_{II} < 0$, and $[\lambda_{II}\lambda_{ee} - \lambda_{Ie}^2] > 0$, I assume $\lambda_{ee} < 0$. See the proof of Proposition 2.2 in Appendix A for detail.

Proposition 2.2. *In the noncommitment case, the optimal general investment I_{nc}^* and the optimal customer-specific investment e_{nc}^* of an HMO are: (1) decreasing with the number of HMOs in the market n ; (2) increasing with the verification cost V ; (3) increasing with a customer's utility of being healthy H ; and (4) decreasing with transferability of preventive care μ .*

Intuitively, having more HMOs in the market increases customers' bargaining power, which reduces both the offer price an HMO can charge and the acceptance probability of a customer. Thus, it reduces HMOs' incentives to invest. When the verification cost is high, it makes the offer price proposed by the original HMO more attractive, which results in a higher profit. Hence HMOs are willing to invest more. Alternatively, customers are willing to pay more if they value more of their health, which makes HMOs more willing to invest more. Finally, a higher transferability of preventive care raises the total surplus when a customer rejects the offer, which in turn reduces both the offer price and the acceptance probability of the customer. Thus, it reduces HMOs' incentives to invest.

Proposition 2.2 indicates the aggravation of the holdup problem between an HMO and any of its customers due to HMO competition. However, it also shows that this problem can be mitigated by a higher verification cost, a higher utility of a customer being healthy, or a lower transferability of preventive care.

One should note that, in fact, an HMO receives extra profits from switched customers of other HMOs. Assume, because of symmetry, that each of other HMOs invests the same amount on e and I , namely $\hat{e}$ and $\hat{I}$, and each HMO has the same

opportunity, which is $1/(n-1)$, to get switched customers, then the ex ante average extra profit of a switched customer of the other HMO is

$$\int_{\hat{e}}^{\hat{e}} \frac{1}{n-1} (1 - \lambda(n)) [H - V - c + \mu\phi(\hat{e})] (1 - a(\hat{O})) dF(c, \hat{I}),$$

where $\hat{e}$ and $\hat{I}$ are investments of other HMOs, and $\hat{O}$ is the offer price made by other HMOs. This extra profit is a constant with respect to variables e , I , and O , assuming that none of $\hat{e}$, $\hat{I}$, and $\hat{O}$ be affected by the HMO's decisions. Therefore, excluding this extra profit does not affect the optimal solutions of e_{nc}^* , I_{nc}^* , and O^* . However, it should be taken into account in an HMO's overall profit when one looks for the optimal uniform preventive care price. This is because price affects demand facing an HMO, which in turn affects the size of the summation of extra profits.

2.4.1.2. The optimal uniform preventive care price

Suppose HMO₀ charges at P and all other HMOs charge at $\hat{P}$ because of symmetry.²⁸ Then, the marginal demander at location $\tilde{x} \in [0, 1/n]$ will be indifferent between HMO₀ and HMO₁, if

$$P + \tilde{x} = \hat{P} + (1/n - \tilde{x}).$$

This is equivalent to say that only the preventive care price and the associated disutility matter to customers, not the subsequent investments or treatment costs, because the $EU_3(I^*, e^*)$ on both sides are canceled out. Thus, the right demand facing HMO₀ is $\tilde{x} = (1/n - P + \hat{P})/2$, and the total demand facing HMO₀ is $2\tilde{x} = (1/n - P + \hat{P})$.

With this demand, the optimal investments, and the optimal expected stage-3 profits, HMO₀'s problem is to select P to

²⁸ The argument used in this subsection is similar to that in Salop (1979).

$$\begin{aligned} \text{Maximize}_P \Pi = & -J + 2 \int_0^{\tilde{x}} \left\{ P - I_{nc}^* - e_{nc}^* + \int_c^{\tilde{x}} \exp\left(\frac{\lambda(c-c)}{G}\right) GdF(c, I_{nc}^*) \right\} dx \\ & + 2 \int_{\tilde{x}}^{1/2} \int_c^{\tilde{x}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(\hat{e})] (1 - \exp\left(\frac{\lambda(c-c)}{G}\right)) dF(c, \hat{I}) dx, \end{aligned}$$

where $\tilde{x} = (1/n - P + \hat{P})/2$, $G(c, n, e, \mu, H, V) \equiv H - (c - \phi(e)) - \lambda(n)[H - V - (c - \mu\phi(e))]$, and

$I_{nc}^* = \hat{I}$ and $e_{nc}^* = \hat{e}$ for symmetry. That is, HMO₀'s problem is to

$$\begin{aligned} \text{Maximize}_P \Pi = & -J + \left\{ P - I_{nc}^* - e_{nc}^* + \int_c^{\tilde{x}} \exp\left(\frac{\lambda(c-c)}{G}\right) GdF(c, I_{nc}^*) \right\} \left(\frac{1}{n} - P + \hat{P} \right) \\ & + \left\{ \int_c^{\tilde{x}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(e_{nc}^*)] (1 - \exp\left(\frac{\lambda(c-c)}{G}\right)) dF(c, I_{nc}^*) \right\} \\ & \times \left(1 - \frac{1}{n} + P - \hat{P} \right) \end{aligned}$$

From the first order condition, the optimal interior solution for P , denoted as P_{nc}^* , is characterized by the following function

$$\begin{aligned} P_{nc}^* = & \frac{1}{2} \left\{ \hat{P} + \frac{1}{n} + I_{nc}^* + e_{nc}^* - \int_c^{\tilde{x}} \exp\left(\frac{\lambda(c-c)}{G}\right) GdF(c, I_{nc}^*) \right. \\ & \left. + \int_c^{\tilde{x}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(e_{nc}^*)] (1 - \exp\left(\frac{\lambda(c-c)}{G}\right)) dF(c, I_{nc}^*) \right\}. \end{aligned}$$

Put $P_{nc}^* = \hat{P}$, because of symmetry, then the optimal preventive care price is

$$\begin{aligned} P_{nc}^* = & \frac{1}{n} - \left[-I_{nc}^* - e_{nc}^* + \int_c^{\tilde{x}} \exp\left(\frac{\lambda(c-c)}{G}\right) GdF(c, I_{nc}^*) \right] \\ & + \int_c^{\tilde{x}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(e_{nc}^*)] (1 - \exp\left(\frac{\lambda(c-c)}{G}\right)) dF(c, I_{nc}^*). \end{aligned} \quad (2.14)$$

Similar to the result of the standard model of monopolistic competition in the circular-city setting, where the equilibrium price is the marginal cost plus $1/n$, equation (2.14) points out that each HMO gives away the profit to be earned at stage three, as shown in the bracket, and charges $1/n$. More interestingly, the last term in the right hand side of equation (2.14) shows that an HMO charges a customer the potential extra profit it

could get later if he originally were a customer of any other HMO. Last, as one can see, the preventive care price may be greater than, equal to, or less than the sum of the investment costs, i.e., $I_{nc}^* + e_{nc}^*$.

2.4.2. The Commitment Case

When HMOs can commit to the levels of the investments announced in the contract, they have to decide the optimal levels of I and e at the beginning of stage 2, together with the optimal decision of the preventive care price P . On the other hand, not only the preventive care price and the associated disutility matter to customers, but also do the subsequent investments and treatment costs.

A customer's utility after bargaining is $\lambda(n)[H-V-c+\mu\phi(e)]$, for illness level c . Ex ante, because he does not know his own level, he uses expected utility, i.e.,

$$\int_{\tilde{c}}^{\bar{c}} \lambda[H-V-c+\mu\phi(e)]dF(c, I), \text{ in considering which HMO to choose. Suppose HMO}_0$$

charges at P and all other HMOs charge at $\hat{P}$ because of symmetry. Then, the marginal demander at location $\tilde{x} \in [0, 1/n]$ will be indifferent between HMO₀ and HMO₁, if

$$\begin{aligned} -P - \tilde{x} + \int_{\tilde{c}}^{\bar{c}} \lambda[H-V-c+\mu\phi(e)]dF(c, I) &= -\hat{P} - \left(\frac{1}{n} - \tilde{x}\right) \\ &+ \int_{\tilde{c}}^{\bar{c}} \lambda[H-V-c+\mu\phi(\hat{e})]dF(c, \hat{I}), \end{aligned}$$

where $\hat{I}$ and $\hat{e}$ are investments made by HMO₁. Thus, the right demand facing HMO₀ is

$$\begin{aligned} \tilde{x} = \frac{1}{2} \left\{ \frac{1}{n} - P + \hat{P} + \int_{\tilde{c}}^{\bar{c}} \lambda[H-V-c+\mu\phi(e)]dF(c, I) \right. \\ \left. - \int_{\tilde{c}}^{\bar{c}} \lambda[H-V-c+\mu\phi(\hat{e})]dF(c, \hat{I}) \right\}, \end{aligned} \quad (2.15)$$

and the total demand facing HMO₀ is $2\tilde{x}$.

With this demand and the optimal expected stage-3 profits, the HMO₀'s problem

is to select P , I , and e to

$$\begin{aligned} \text{Maximize}_{P,I,e} \Pi = & -J + 2 \int_0^{\tilde{x}} \left\{ P - I - e + \int_{\underline{e}}^{\tilde{e}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) G dF(c, I) \right\} dx \\ & + 2 \int_{\tilde{x}}^{\hat{x}^{1/2}} \int_{\underline{e}}^{\tilde{e}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(\hat{e})] (1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) dF(c, \hat{I}) dx, \end{aligned}$$

where $G(c, n, e, \mu, H, V) \equiv H - (c - \phi(e)) - \lambda(n)[H - V - (c - \mu\phi(e))]$, and $\tilde{x}$ is stated in equation (2.15). That is, the HMO₀'s problem is to

$$\begin{aligned} \text{Maximize}_{P,I,e} \Pi = & -J + \left\{ P - I - e + \int_{\underline{e}}^{\tilde{e}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) G dF(c, I) \right\} \times \left(\frac{1}{n} - P + \hat{P} + \right. \\ & \left. \int_{\underline{e}}^{\tilde{e}} \lambda [H - V - c + \mu\phi(e)] dF(c, I) - \int_{\underline{e}}^{\tilde{e}} \lambda [H - V - c + \mu\phi(\hat{e})] dF(c, \hat{I}) \right) \\ & + \left\{ \int_{\underline{e}}^{\tilde{e}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(\hat{e})] (1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) dF(c, \hat{I}) \right\} \times \\ & \left(1 - \frac{1}{n} + P - \hat{P} - \int_{\underline{e}}^{\tilde{e}} \lambda [H - V - c + \mu\phi(e)] dF(c, I) \right. \\ & \left. + \int_{\underline{e}}^{\tilde{e}} \lambda [H - V - c + \mu\phi(\hat{e})] dF(c, \hat{I}) \right). \end{aligned}$$

Solving for the first order conditions with respect to P , I , and e , respectively, and then putting $P_c^* = \hat{P}$, $I_c^* = \hat{I}$, and $e_c^* = \hat{e}$, because of symmetry, I get the following functions characterizing the optimal interior solutions.²⁹

$$\begin{aligned} P_c^* = & \frac{1}{n} - \left[-I_c^* - e_c^* + \int_{\underline{e}}^{\tilde{e}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) G dF(c, I_c^*) \right] \\ & + \int_{\underline{e}}^{\tilde{e}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(e_c^*)] (1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) dF(c, I_c^*), \\ \lambda_I = & \int_{\underline{e}}^{\tilde{e}} \left\{ \exp\left(\frac{\lambda(n)(\underline{c}-c)}{G}\right) G + \lambda [H - V - c + \mu\phi(e)] \right\} dF_I(c, I) - 1 = 0, \end{aligned} \quad (2.16)$$

²⁹ To distinguish from the noncommitment case, I denote as P_c^* , I_c^* , and e_c^* , respectively, the optimal levels of P , I , and e in the commitment case.

or, by using integration by parts,

$$\lambda_I \equiv \int_{\underline{c}}^{\bar{c}} F_I(c, I) \left\{ \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) [1 - (1-\lambda)\frac{\lambda(\underline{c}-c)}{G}] + \lambda \right\} dc - 1 = 0, \quad (2.17)$$

$$\lambda_e \equiv \int_{\underline{c}}^{\bar{c}} \left\{ \exp\left(\frac{\lambda(n)(\underline{c}-c)}{G}\right) (1 - \lambda\mu)\phi'(e) \left(1 - \frac{\lambda(\underline{c}-c)}{G}\right) + \lambda\mu\phi' \right\} dF - 1 = 0. \quad (2.18)$$

Note that, first, the optimal preventive care price in the commitment case has the same solution form as that in the noncommitment case. However, as one will see later, it does not mean that the two optimal preventive care prices are the same. Second, as in the noncommitment case, the optimal preventive care price may be greater than, equal to, or less than the total of the investment costs, i.e., $I_c^* + e_c^*$.

Third, as in the noncommitment case, from equations (2.17) and (2.18), it is also hard to find closed-form solutions for the optimal general investment I_c^* and the optimal customer-specific investment e_c^* , unless a more specific structure of the model is introduced. Nonetheless, this does not affect the understanding of comparative statics, which is reported in the following proposition.

Proposition 2.3. *In the commitment case, the optimal general investment I_c^* and the optimal customer-specific investment e_c^* of an HMO: (1) are ambiguous in the direction of changes with the number of HMOs in the market n ; (2) increase with the verification cost V ; (3) increase with a customer's utility of being healthy H ; and (4) are ambiguous in the direction of changes with transferability of preventive care μ .*

Intuitively, if a customer is concerned about investment levels when choosing his original HMO, an HMO will take into account the effect of investment levels on the customer's utility when determining its optimal investments. This can be clearly seen in

the terms $\int_c^{\bar{c}} \lambda [H - V - c + \mu \phi(e)] dF_I$ in equation (2.17) and $\lambda \mu \phi'$ in equation (2.18),

which are differentiations of a customer's expected stage-3 utility with respect to I and e , respectively. The basic idea is, an HMO is concerned not only about its expected stage-3 profit, but also about whether it will lose business at stage 2. If the effect of losing business is stronger in the different direction than the effect of earning profit, the results of comparative statics in the commitment case will upset those in the noncommitment case, which are shown in Proposition 2.3.

2.4.3. Comparisons With The Social Optimal Investments

It would be interesting to compare the optimal investment levels in both cases with the social optimal investment level. This subsection considers this issue. Before proceeding, I need to analyze the social planner's problem, which is to select I and e to

$$\text{Maximize } \int_c^{\bar{c}} [H - (c - \phi(e))] dF(c, I) - e - I.$$

The interior social optimal investments, denoted as I^{**} and e^{**} , are characterized by the following first-order conditions

$$\lambda_I \equiv \int_c^{\bar{c}} [H - (c - \phi(e))] dF_I(c, I) - 1 = 0,$$

or, by using integration by parts,

$$\lambda_I \equiv \int_c^{\bar{c}} F_I(c, I) dc - 1 = 0, \quad (2.19)$$

$$\lambda_e \equiv \int_c^{\bar{c}} \phi'(e) dF(c, I) - 1 = 0, \text{ or } \phi'(e) - 1 = 0. \quad (2.20)$$

Second, recall that the optimal investment levels, I_{nc}^* and e_{nc}^* , in the noncommitment case are characterizing in equations (2.11) and (2.12). In addition, the

optimal investment levels, I_c^* and e_c^* , in the commitment case are characterizing in equations (2.16) and (2.17). Then, the following proposition for comparison yields.

Proposition 2.4. (1) $e_{nc}^* < e_c^* < e^{**}$; (2) $I_{nc}^* < I_c^*$; (3) $I_{nc}^* < I^{**}$; (4) $I_c^* > I^{**}$ if

$$\left\{ \exp\left(\frac{\lambda(\bar{c}-\bar{c})}{G(\bar{c})}\right)[1 - (1-\lambda)\frac{\lambda(\bar{c}-\bar{c})}{G(\bar{c})}] + \lambda \right\} > 1.$$

Intuitively, since an HMO could not fully get the benefit of its investments, it has no incentive to invest up to the social optimal levels. This underinvestment phenomenon can be seen clearly in the noncommitment case, and is the famous holdup problem recognized in Williamson (1975, 1985). When HMOs can commit to investments, since customers will value the benefits of those investments, this business-stealing effect gives HMOs an incentive to invest more. Hence, the optimal investment levels in the commitment case are higher than those in the noncommitment case. In particular, if the business-stealing effect is sufficiently large, HMOs may invest more than socially desired, i.e., the overinvestment phenomenon may occur. However, Proposition 2.4 indicates that, although HMOs may overinvest in the general investment I , they only underinvest in the customer-specific investment e . This can be explained by the differences in the natures of these two investments. While the general investment is applied to the whole population, the customer-specific investment is only applied to one specific customer. An HMO will totally lose the benefit of the customer-specific investment if the customer switches to another HMO. Therefore, it is not likely that HMOs will overinvest in the customer-specific investment.

2.4.4. The Comparison Of The Two Optimal Preventive Care Prices

Furthermore, it can be noted from Proposition 2.4 that, although the solution for P_{nc}^* in equation (2.13) has the same functional form as that for P_c^* in equation (2.15), P_{nc}^* may not be equal to P_c^* , because the optimal investment levels in both cases are not the same. In fact, it can be proved that $P_{nc}^* < P_c^*$, given two sufficient conditions.

Proposition 2.5. $P_{nc}^* < P_c^*$, given that $0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} \leq 1$ and $2\lambda-1-\lambda^2\mu < 0$.

Proposition 2.5 presents two conditions that are jointly sufficient for P_{nc}^* to be smaller than P_c^* . The first condition, $0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} \leq 1$, implies $\lambda > 1/2$.³⁰ In other words, the bargaining power of customers must be no lower than that of HMOs. The second condition, $2\lambda-1-\lambda^2\mu < 0$, restricts the range of μ for each specific λ . Figure 2.3. shows the nonempty set of (λ, μ) satisfying these two conditions.

2.5. Free Entry And Social Optimum

Another concern is, if the healthcare market is open to all potential HMOs, will there be too many or too few HMOs entering into the market from the perspective of a society. From the analysis of sections 2.3 and 2.4, more HMOs entering into the market can reduce customers' disutility, which is beneficial to the society. On the other hand, more HMOs entering into the market will worsen the holdup problem that reduces HMOs' incentives of investing in preventive care and in turn results in higher future

³⁰ To see this, note that $1 < \frac{H-(c-\phi(e))}{G}$, because $0 < G = H-(c-\phi(e))-\lambda[H-V-(c-\mu\phi(e))] < H-(c-\phi(e))$. Then, $\frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} \leq 1 < \frac{H-(c-\phi(e))}{G}$, or $\frac{1-\lambda}{\lambda} < 1$. Therefore, $\lambda > 1/2$.

medical costs to the society. This is in addition to the generally high setup costs.³¹ Thus, it is not clear whether more HMOs enter the market than is socially desired. This section attempts to answer this important question and get policy implications.

The structure of the model is kept the same as described in section 2.2, except that two situations are considered in stage 1. One is free entry, and the other is the social optimum. Note that the social optimum here refers to the situation in which the regulator only controls the number of HMOs in the market, rather than both the number and investments. Again, there are two cases regarding HMOs' credibility in committing to the investment levels announced in the contract.

2.5.1. The Noncommitment Case

First, in the free entry, firms can enter and exit the market freely, which implies that a firm will enter the market until its net overall profit is 0. Overall, a customer's ex ante expected utility is

$$\begin{aligned} \Pi^c = & -x - \frac{1}{n} - I_{nc}^* - e_{nc}^* + \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) G dF(c, I_{nc}^*) \\ & + \int_{\underline{c}}^{\bar{c}} [H - V - c + \mu\phi(e_{nc}^*)] \left[\lambda - \frac{1-\lambda}{n-1} (1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) \right] dF(c, I_{nc}^*), \end{aligned} \quad (2.21)$$

and an HMO's overall expected profit, denoted as Π_n to mean n HMOs in the market, is

$$\Pi_n = -J + \frac{1}{n^2} + \int_{\underline{c}}^{\bar{c}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(e_{nc}^*)] (1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) dF(c, I_{nc}^*). \quad (2.22)$$

Thus, in equilibrium, the number of HMOs, denoted by n^e , entering the market is determined by $\Pi_{n^e} = 0$.

³¹ According to Winslow (1995), the American Medical Association estimates that setup costs for an HMO may exceed \$15 millions.

One may wonder if an HMO's overall expected profit always decreases with the number of HMOs, i.e., if $d\Pi_n/dn < 0$. Unfortunately, this may not be always true. Lemma 2.1 finds conditions that are jointly sufficient for it to be true.

Lemma 2.1. *In the noncommitment case, an HMO's overall expected profit decreases*

with the number of HMOs, given that $0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} \leq 1$, $2\lambda-1-\lambda^2\mu < 0$, and

$\partial\Pi_n/\partial I > 0$.

Lemma 2.1 states that, given the three conditions, each HMO gets higher profits if fewer HMOs are in the market. In other words, firms benefit from relaxed competition. With fewer HMOs in the market, customers' disutilities rise so as to enhance the monopoly power of their original HMOs. This gives HMOs a chance to extract more surplus from customers which leads to higher profits. Alternately, with fewer HMOs in the market, HMOs invest more in preventive care, which can also lead to higher profits.

Although Lemma 2.1 seems very intuitive, it justifies the concerns from the FTC and the Department of Justice that the motivation of consolidations of HMOs is, instead of achieving efficiencies, to gain market power which could result in an increase in premiums.³² In fact, this may not be true. In the model, the premium, which is the offer price $O^*(c) = H - \lambda[H - V - c + \mu\phi(e)]$, depends both negatively on a customer's bargaining power, which increases with n , and on the benefit from the investment in preventive care, which decreases with n . Although the consolidation of HMOs decreases a customer's

³² The study of Feldman et al. (1999) finds that earlier HMO mergers may have caused only a small increase in the type of local market concentration that may increase prices, later mergers may be motivated by considerations of increasing local market concentration. Also, see Iglehart (1998) for the role the Federal Trade Commission plays in the health care market.

bargaining power, resulting in a higher offer price, it also increases the benefit of investment, resulting in a lower offer price. Therefore, it is not clear if premiums will rise after consolidations of HMOs. One can even see a decrease in premiums if the benefit effect dominates the bargaining power effect. This suggests the need of more consideration in preventing HMOs consolidation.

Secondly, in the social optimum, the regulator limits the number of firms entering the market so that social welfare is maximized.³³ The social welfare, denoted by SW , is given by

$$\begin{aligned} SW &\equiv CS + PS \\ &= n\Pi_n - \frac{5}{4} \frac{1}{n} - I^* - e^* + \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c} - c)}{G}\right) G dF(c, I^*) \\ &\quad + \int_{\underline{c}}^{\bar{c}} [H - V - c + \mu\phi(e^*)] \left[\lambda - \frac{1 - \lambda}{n - 1} (1 - \exp\left(\frac{\lambda(\underline{c} - c)}{G}\right)) \right] dF(c, I^*), \end{aligned} \quad (2.23)$$

where Π_n is given by equation (2.21), $I^* = I_{nc}^*$, and $e^* = e_{nc}^*$.³⁴

The goal of the regulator is to choose the optimal number of HMOs, denoted as n^* , to maximize the social welfare SW . Thus, the solution for n^* is characterized by the first-order condition $SW'(n^*) = 0$, assuming that the second-order condition, $SW''(n^*) < 0$, is satisfied.

Before I compare the optimal number of HMOs in the free entry market with the social optimum, an intermediate lemma is needed as follows.

Lemma 2.2. $SW'(n) < \Pi_n$ given $0 < \frac{1 - \lambda}{\lambda} \frac{H - (c - \phi(e))}{G} \leq 1$, $2\lambda - 1 - \lambda^2\mu < 0$, and $\partial\Pi_n/\partial I > 0$.

³³ One can follow Salop (1979) to get the optimal number of firms by minimizing social cost.

³⁴ This essay only considers equal weight on both sides of health plans.

With Lemma 2.1 and Lemma 2.2, consequently, I get the result that there are too many HMOs in the free entry market, compared with the social optimum.

Proposition 2.6. *Given $0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} \leq 1$, $2\lambda-1-\lambda^2\mu < 0$, and $\partial\Pi_n/\partial l > 0$, (1)*

HMOs are profitable in the social optimum. That is, $\Pi_{n^*} > 0$; (2) there are too many

HMOs in the free-entry regime. That is, $n^* < n^e$.

Intuitively, a large sharing proportion λ for customers means a small probability for customers to accept offers from their original HMOs. In other words, HMOs are more likely to have business with switched customers, which results in a large extra profit. Although the extra profit is positively related with other HMOs' investments, every HMO is still willing to make high investments because all HMOs invest the same amounts in equilibrium. Further, combined with a large transferability of preventive care μ , HMOs will save a lot in future treatment costs. Therefore, each firm's profit is positive in the social optimum. This positive profit attracts more HMOs to enter when the market allows entry freely. Then, every HMO's profit decreases not only because, with a even large sharing proportion, it can only extract a even smaller surplus, but also because the decrease of investments due to competition.

A policy implication from Proposition 2.6 is that, the HMO industry should be regulated appropriately, such as taxing a fixed fee per firm on entry to raise entry barriers, because free entry leads to social inefficiency under some circumstances. Restricting the number of HMOs entering the market may increase customers' disutilities and allow HMOs to earn positive economic profit. However, it gives HMOs incentives to invest properly in preventive care, which can reduce social resources allocated in the future medical expense and can be much more beneficial to the society.

2.5.2. The Commitment Case

The analysis for the commitment case is not very different from that for the noncommitment case. Hence, below, I only give the relevant results without explanations. First, a customer's ex ante overall expected utility is

$$\begin{aligned} \Pi^e = & -x - \frac{1}{n} - I_c^* - e_c^* + \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) G dF(c, I_c^*) \\ & + \int_c^{\bar{c}} [H - V - c + \mu\phi(e_c^*)] \left[\lambda - \frac{1-\lambda}{n-1} (1 - \exp\left(\frac{\lambda(c-c)}{G}\right)) \right] dF(c, I_c^*), \end{aligned} \quad (2.24)$$

and an HMO's overall expected profit, denoted as Π_n to mean n HMOs in the market, is

$$\Pi_n = -J + \frac{1}{n^2} + \int_c^{\bar{c}} \frac{1-\lambda}{n-1} [H - V - c + \mu\phi(e_c^*)] (1 - \exp\left(\frac{\lambda(c-c)}{G}\right)) dF(c, I_c^*), \quad (2.25)$$

which have the same forms as in the noncommitment case. Thus, in the free-entry equilibrium, the number of HMOs, denoted by n^e , entering the market is determined by $\Pi_{n^e} = 0$.

Like in the noncommitment case, it is not always true that an HMO's overall expected profit decreases with more HMOs in the market, i.e., if $d\Pi_n/dn < 0$. Lemma 2.3 finds conditions that are jointly sufficient for it to be true.

Lemma 2.3. *In the commitment case, an HMO's overall expected profit decreases with the number of HMOs in the market, given that $0 < \frac{1-\lambda}{\lambda} \frac{H - (c - \phi(e))}{G} \leq 1$, $2\lambda - 1 - \lambda^2 \mu < 0$,*

$\partial\Pi_n/\partial I > 0$, $dI/dn < 0$, and $de/dn < 0$.

Lemma 2.3 presents five conditions that are jointly sufficient for $d\Pi_n/dn < 0$ to hold. The first three conditions are the same as in Lemma 2.1, and the last two conditions are to make sure that the optimal investment levels decrease with the number of HMOs.

Secondly, in the social optimum, the regulator limits the number of firms entering the market so that social welfare is maximized. The social welfare, denoted by SW, is given by

$$\begin{aligned}
 SW &\equiv CS + PS \\
 &= n\Pi_n - \frac{5}{4} \frac{1}{n} - I^* - e^* + \int_{\underline{e}}^{\bar{e}} \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) GdF(c, I^*) \\
 &\quad + \int_{\underline{e}}^{\bar{e}} [H - V - c + \mu\phi(e^*)] \left[\lambda - \frac{1-\lambda}{n-1} (1 - \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right)) \right] dF(c, I^*),
 \end{aligned} \tag{2.26}$$

where Π_n is given by equation (2.23), $I^*=I_e^*$, and $e^*=e_e^*$.

The goal of the regulation is to choose the optimal number of HMOs, denoted as n^* , to maximize the social welfare SW. Thus, the interior solution for n^* is characterized by the first-order condition $SW'(n^*) = 0$, assuming that the second-order condition, $SW''(n^*) < 0$, is satisfied.

An intermediate lemma is needed as follows before comparing the equilibrium number in the free entry with the social optimum.

Lemma 2.4. $SW'(n) < \Pi_n$ given $0 < \frac{1-\lambda}{\lambda} \frac{H - (c - \phi(e))}{G} \leq 1$, $2\lambda - I - \lambda^2\mu < 0$, $\partial\Pi_n/\partial I > 0$, $dI/dn < 0$, and $de/dn < 0$.

Consequently, comparing the equilibrium number of firms, n^e , in the free-entry with the social optimum, n^* , I get the following results.

Proposition 2.7. Given $0 < \frac{1-\lambda}{\lambda} \frac{H - (c - \phi(e))}{G} \leq 1$, $2\lambda - I - \lambda^2\mu < 0$, $\partial\Pi_n/\partial I > 0$, $dI/dn < 0$,

and $de/dn < 0$, (1) HMOs are profitable in the social optimum. That is, $\Pi_{n^*} > 0$; (2) there are too many HMOs in the free-entry regime. That is, $n^* < n^e$.

The intuition and policy implication are the same as in section 2.5.1.

2.6. Conclusion

Vigorous competition among HMOs has been viewed as a force to curb the two-digit increase in annual health care premiums. This essay, however, sheds light on a very likely negative relationship between competition and investments of HMOs in preventive care, which is often claimed to be an important characteristic of HMOs. Moreover, this essay discusses numbers of HMOs in the free entry and in the social optimum as a policy implication.

The principal findings are as follows. First, there is a unique separating equilibrium in which HMOs offer each customer a premium that is linear and increasing with his exact illness level, and each customer, without information of his own illness level, accepts the offer with a probability that is nonincreasing with the premium at a decreasing rate. Second, generally, there is a hold-up problem, i.e., underinvestments, for HMOs, which can be mitigated by a higher verification cost, a higher utility of a customer being healthy, or a lower transferability of preventive care. However, too much general investment, which is used to reduce the probability of customers being severely ill, can occur when HMOs can commit to the investment levels announced in the plan, due to the business-stealing effect arising from customers valuing investments. Third, HMOs can be profitable in the social optimum, which leads to too many HMOs entering in the free entry market.

This essay has focused on capturing the current situation to examine the effect of competition for already enrolled customers on the holdup problem facing an HMO. From the policy perspective, it is possible to solve the holdup problem by adding special

mechanism designs. Many papers have offered good insights on this issue in alternative circumstances.³⁵

Also, this essay highlights the effect of competition on underinvestments of HMOs by assuming interactions between HMOs and many customers, which represents many small employers. For larger employers with higher bargaining power, it is possible that they will renegotiate with their original HMOs before they switch their employees to other HMOs. In such case, one can incorporate renegotiation design³⁶ in the model and reexamine the issues discussed in this essay.

Moreover, this essay raises a need of an empirical study in the relationship between HMO competition and HMO investments in preventive care.

Finally, in order to simplify the model, this essay has treated an HMO as a provider of both health insurance and health care services. This means a group- or staff-model HMO. In practice, there are two other levels of HMOs: network model and individual practice association (IPA) model, which also need a careful investigation on issues discussed in this essay.

³⁵ See Chung (1991), Edlin and Reichelstein (1996), Macleod and Malcomson (1993), Noldeke and Schmidt (1995), and Regerson (1992) for alternative solutions to the holdup problem under different circumstances.

³⁶ See Aghion, Dewatripont, and Rey (1994), and Noldeke and Schmidt (1995) for renegotiation design.

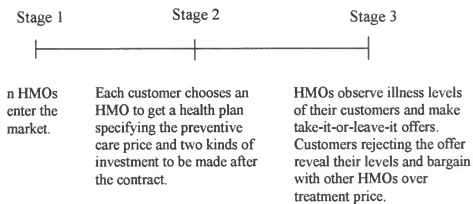


Figure 2.1. The Time Line Of The Model

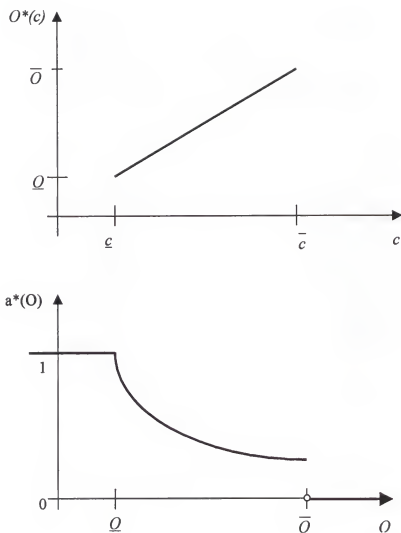


Figure 2.2. The Equilibrium Strategies ($O^*(c)$ And $a^*(O)$) Of The HMO And The Customer

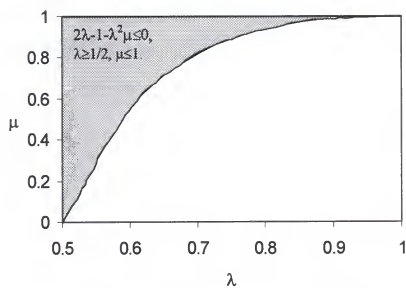


Figure 2.3. The Area Of (λ, μ) Satisfying $2\lambda - 1 - \lambda^2\mu \leq 0$, $\lambda \geq 1/2$, And $\mu \leq 1$

CHAPTER 3 PRICES AND MONEY-BACK GUARANTEES AS A SIGNAL OF PRODUCT QUALITIES IN DUOPOLY

3.1. Introduction

Consumers often cannot determine with certainty before making their purchases the difference in qualities of products sold by different firms or at different stores. For instance, a consumer may read newspapers or consumer reports analyzing differentiated products a long time before purchase. When he actually shops and purchases the good, he may only remember that there are one high-quality good and one low-quality good, instead of each product's exact quality. Alternatively, many products can be imitated by "pirates" in such a way that consumers cannot tell stores selling the fake from stores selling the authentic product. In these situations, consumers have to form beliefs about which product to buy without having complete information on product quality.

The question arises: can price alone serve as a signal of product quality for consumers? If so, consumers can have confidence about their purchases. If not, is there any instrument that, together with price, can signal product quality?¹

This essay studies the informational role of price and money-back guarantees (or refunds) in the framework of duopoly with quality uncertainty for risk-neutral

¹ In the literature of repeat purchases, a low introductory price can serve as a signal of high quality if the difference in gain due to repeat purchases exceeds the cost advantage enjoyed by the low-quality firm. Obviously, it is a costly signal. Moreover, the repeat-purchase argument does not work in a one-shot game where firms and consumers meet only once. As discussed in Kirmani and Rao (2000), repeat purchase is unnecessary for such cost-risking signals as warranties and money-back guarantees.

consumers.² Specifically, I study two cases. The first case, inspired by Shieh (1996), considers homogeneous consumers with the same valuation for a working product, and two vertically differentiated firms with different unit costs of production, each of which simultaneously decides a refund policy followed by a price. The second case, inspired by Hertzendorf and Overgaard (1998a, 1998b), considers heterogeneous consumers with different valuation for a working product, and two vertically differentiated firms with the same unit costs of production, each of which simultaneously decides a refund policy and a price.

In the first case, I show that price alone cannot signal product quality. Therefore, differentiated firms pool together at a price as high as possible according to the system of consumer beliefs. An outcome consistent with implicit collusion can arise. Second, when each firm can choose whether or not to offer refunds before the price-setting game, the unique separating equilibrium, which can exist depending on parameters, is that only the high-quality firm offers refunds. This result shows that the refund itself is a signal of quality, contrasting with the result of Shieh (1996), where the refund together with price is a signal. However, unlike the monopoly case in Shieh (1996) and Levy (1997), signaling is costly in the sense that the high-quality firm cannot regain its complete-information price and profits. Therefore, firm H may not find it in its best interest to separate from firm L by offering refunds should it cost firm H too much. Surprisingly, consumers, who lack information, benefit from the separation of the strategies of the two firms. Moreover, as in Shieh (1996), social welfare is restored to the complete-

² There are several roles which warranties and guarantees can play. The first role is risk-sharing, e.g., Heal (1977). The second role is information transmission, e.g., Spence (1977), and Grossman (1981).

information level if the high-quality firm separates from the low-quality firm. As for the low-quality firm, it does not offer a refund in a separating equilibrium, but it may offer it in a pooling equilibrium.

In the second case, following the definition of sequential equilibrium proposed in Hertzendorf and Overgaard (1998a, 1998b), I show that none of the complete information equilibrium profiles can constitute a separating equilibrium when incomplete information is present. Then, I illustrate the existence of separating sequential equilibria for a sufficiently large degree of vertical differentiation. Specifically, I assume as 0.9 the working probability of the high-quality good, π_H , then separating equilibria exist for firm L offering no refund if the working probability of the low-quality good, π_L , is less than 0.1561. In the case where firm L offers a refund, separating equilibria exist for $\pi_L < 0.6$. Compared to the no-refund case, enabling the choice of the refund policy expands the range of the parameter π_L for the existence of separating equilibria. However, it depends on the refund policy chosen by firm L, instead of firm H. Furthermore, I identify the set of multiple pooling sequential equilibria for any degree of vertical differentiation.

Subsequently, I discuss the refinements of separating and pooling equilibria, respectively. Following Hertzendorf and Overgaard (1998b), I reduce the set of multiple separating equilibria based on the refinement of REDE (Resistance of Equilibrium Defections). I may or may not obtain a unique refined separating equilibrium depending on the parameter π_L . Specifically, there are two equilibria for $\pi_L < 0.1561$, namely, $(\hat{R}_H, \hat{R}_L) = (0, 0)$ and $(0, 1)$, and only one equilibrium for $\pi_L \in (0.1561, 0.6)$, namely, $(\hat{R}_H, \hat{R}_L) = (0, 1)$ for $\pi_L \in (0.1561, 0.3)$ and $(1, 1)$ for $\pi_L \in (0.3, 0.6)$. More surprisingly, the case of $(\hat{R}_H, \hat{R}_L) = (0, 1)$ can be a separating equilibrium for a sufficiently large

degree of vertical differentiation. This result seems contradictory to one's intuition and contrasts with the homogeneous-consumer case, where $(1, 0)$ is the only separating equilibrium if one exists. It is so because firm L is indifferent between offering a refund and not, as long as $[1 - (1 - \pi_L)R_L]P_L = (\pi_L / (2\pi_H)) [1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H$. But, the best reply of firm H to firm L offering no refund is offering no refund either. Therefore, $(1, 0)$ is not an equilibrium. Intuitively, offering refunds incurs additional cost, and so is there a cost for signaling by distorting the price. For firm H, the sum of the refund cost and the signaling cost for $(1, 0)$ outweighs that for $(0, 0)$ so that it would rather choose not to offer refunds. As for the set of pooling sequential equilibria, I demonstrate that no pooling equilibrium survives the refinement of the incentive argument for any parameter π_L .

The result of separating equilibria shows that the refund itself can be a signal of product quality for $\pi_L \in (0.1561, 0.3)$, but not otherwise. This partially contrasts with the result in Shieh (1996), where the refund must be together with price to signal product quality in monopoly. Finally, comparing the equilibrium prices with the two information structures, I find that the equilibrium prices under incomplete information are distorted upwards. Therefore, signaling via money-back guarantees in duopoly is costly, which is not inconsistent with the homogeneous-consumer case but contrasts with Shieh (1996) and Levy (1997), where signaling in monopoly is costless. However, consumers are weakly worse off under incomplete information, contrasting with the homogeneous-consumer case.

This essay is directly related to the literature on the signaling of product quality. Many instruments have been discussed as a signal of product quality in monopoly, such

as advertising, e.g., Milgrom and Roberts (1986), warranties, e.g., Emons (1988), and Lutz (1989), money-back guarantees, e.g., Moorthy and Srinivasan (1995), and Shieh (1996), and best-price guarantees, e.g., Levy (1997). The discussion of the case of monopoly is justified by the introduction of a new product. This cannot explain whether or not the high-quality firm should signal its quality in duopoly.

Few papers address the signaling of product quality in duopoly.³ The first paper, by Gal-Or (1989) discusses the signaling role of warranties in duopoly. She considers Cournot competition where firms offer short or long warranty duration as a one-dimensional signal of product quality. Under a system of consumer beliefs, she shows that warranties are perfect signals only in cases where the intrinsic attributes of products are neither too clustered nor too widely spaced. Otherwise, multiple separating equilibria arise, and the informational content of warranties is limited. Under that system of consumer beliefs, no pooling equilibrium is sustained.

Daughety and Reinganum (1997) examine the influences of price competition on a signaling game. However, in their model the two firms produce goods with the same quality and consumers are just uncertain about which quality the two firms produce. In contrast, my model deals with one firm producing goods with high quality and the other low quality. Consumers are uncertain about which firm produces goods with which quality.

³ Gabszewicz and Grilo (1992) analyze the properties of price equilibria in a differentiated duopoly with consumer uncertainty about product quality, which is close to the purpose of this essay. However, consumers in their model form their beliefs before observing prices, namely, exogenous beliefs. Thus, it is not a signaling model. The other paper dealing with duopoly and quality is Crampes and Hollander (1995), that is not a signaling model, either.

Hertzenndorf and Overgaard (1998a) discuss the signaling role played by prices in duopoly. They propose a notion of sequential equilibrium and refinements of sequential equilibria in duopoly and find that separation is only possible when the quality difference of the two products is sufficiently large. Moreover, in equilibrium the prices of both firms are likely distorted above their complete information levels, which implies the costly signaling of prices.

Using the similar approach developed in the former paper, Hertzenndorf and Overgaard (1998b) explore the signaling role played by price and dissipative advertising in duopoly. They find that the introduction of dissipative advertising ensures the existence of separating equilibria for any degree of vertical differentiation. The refinement of REDE reveals the unique separating equilibrium as that only firm H uses advertising when products are sufficiently close substitutes, and that neither firm uses it otherwise. Furthermore, the prices of both firms may be distorted upwards or downwards from the complete information prices depending on the degree of differentiation.

Methodologically, the heterogeneous-consumer case employs a model similar to that in Hertzenndorf and Overgaard (1998b) to discuss the signaling role played by money-back guarantees. These two models are different in essence. Dissipative advertising acts as a fixed cost and does not change the demand function at all. While the money-back guarantee acts as a constant marginal cost and may affect the demand function. Thus, the analysis with the money-back guarantee is not totally the same as that with advertising. For instance, with advertising the unique best reply of firm L is not to advertise. While with refunds the best reply of firm L is any set of (P_L, R_L) such that $[1 - (1 - \pi_L)R_L]P_L = (\pi_L / (2\pi_H)) [1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H$, which yields multiple solutions for (P_L, R_L)

and complicates the analysis. For this reason, I only consider two extreme refund policies, namely, $R=0$ and $R=1$. Furthermore, the results of these two models are not all the same, either. For example, in their model the separating equilibrium is such that $(\hat{A}_H, \hat{A}_L)=(0, 0)$ for sufficiently small parameter π_L and $(1, 0)$ otherwise. Moreover, in their model the unique pooling equilibrium surviving the incentive argument is $(\hat{P}, \hat{A})=(0, 0)$, while in my model no pooling equilibrium surviving the same argument.

The homogeneous-consumer case is inspired by and uses a model similar to Shieh (1996). The main difference is that she discusses the monopoly case with heterogeneous consumers, while I discuss the duopoly case with homogeneous consumers. After the interaction between the two firms is allowed in my model of signaling in duopoly, it becomes more complicated to derive the equilibrium outcomes and the results are not all the same as in Shieh (1996). Nor do the results in the heterogeneous-consumer the same as in Shieh (1996). This shows the inappropriateness of direct application of monopoly to duopoly.

Technically, the analysis of this first case is complicated. As mentioned in Gal-Or (1989), a difficulty arises for analysis of Bertrand competition because the Bertrand equilibria generate discontinuous payoff functions at points where one firm undercuts the price of the other firm in order to eliminate the other firm's market. To reduce the complexity by a little bit, this essay only considers homogeneous consumers, as is in Daughety and Reinganum (1997). Moreover, this assumption enables one to see both the incentive and the ability of the high-quality firm to signal its quality under the most intense price competition.

Finally, since the main feature of the models considered in this essay is multiple senders and one receiver, it is not appropriate to immediately apply the notion of sequential equilibrium suggested by Kreps and Wilson (1982). Nor is it appropriate to apply directly the formal refinements of equilibrium used in the standard one-sender-and-one-receiver signaling game, such as the Intuitive Criterion by Cho and Kreps (1987) and the Criterion of Universal Divinity by Banks and Sobel (1987). Thus, in the spirit of equilibrium notions for a standard signaling game, I shall first present for each game a definition of sequential equilibrium which is similar to Hertzendorf and Overgaard (1998a, 1998b), then propose selection procedures to delete nonsensible equilibria. Specifically, in the first case, no refinement is used for reducing the set of separating sequential equilibria, while different out-of-equilibrium beliefs for upward and downward deviations is allowed and the Pareto-Perfection argument is used for reducing the set of pooling sequential equilibria.⁴ In the second case, following Hertzendorf and Overgaard (1998b), I use REDE (resistant to equilibrium defection) to refine separating sequential equilibria and employ their incentive argument to refine the set of pooling sequential equilibria.

The remainder of the essay is organized as follows. Section 3.2. studies the first case, and Section 3.3. analyzes the second case. In each case, the complete information

⁴ Allowing different beliefs for upward and downward deviation, which is very natural, is suggested by Jonathan Hamilton and Steven Slutsky. A similar setup can be seen in Grossman and Horn (1988), in which model consumers believe that the firm offers high quality if its capacity is above average and vice versa. The Pareto-Perfection argument, suggested in Hertzendorf and Overgaard (1998a), can be viewed as the focal outcome. See Kreps (1990, p.415) for more detail of the focal outcome.

equilibrium is compared to the incomplete information equilibrium. Finally, Section 3.4. presents conclusion and extensions.

3.2. Homogeneous Consumers, Different Unit Costs and Different Timing of Firm Decisions

This section studies the first case, where consumers have the same valuation on a working product and each firm with different unit costs of production simultaneously decides a refund policy followed by a price.

3.2.1. The Basic Model

Consider a four-stage model in which two firms, denoted as firm 1 and firm 2, produce the same good but of different qualities. Either firm's product may immediately break down after purchase.⁵ The quality of each product is therefore simply assumed as the probability that it works, which is π_H for the high-quality good, denoted as good H, and π_L for the low-quality good, denoted as good L, where $0 < \pi_L < \pi_H < 1$. Product qualities and costs are privately informed to the two firms, but not to consumers. Moreover, consumers only know that there is exactly one firm for each type of quality and that two specific unit costs are associated with different qualities, but they do not know whether firm 1 or firm 2 produces the high-quality good. As described in the introduction, this situation can occur for a consumer who read newspapers or consumer reports analyzing differentiated products long time before he really makes the purchase so that he may not be able to match the quality with each good at the time of purchase.

⁵ Alternatively, I can interpret this as the probability of consumer satisfaction with the product purchased, as defined in Milgrom and Roberts (1986). This will, however, induce a resale problem and will complicate the analysis. Hence, in this essay I confine discussion to the interpretation mentioned in the text.

Each firm offers its product to consumers with a contract $\Gamma_q = (P_q, R_q) \in \mathbb{R}_+ \times \{0, 1\}$, $q \in \{H, L\}$, where the price is P_q and the refund policy is R_q , the fraction of purchases which are refunded if the product breaks down. Hence, $R_q P_q$ denotes the size of the refund for product q .⁶ To simplify the analysis, the current essay only considers two extreme refund policies, $R_q = 0$ or $R_q = 1$.⁷ Note that under incomplete information consumers cannot immediately identify Γ_H and Γ_L as contracts offered by firm H and firm L, respectively.

Consumers are identical in their monetary valuation of a working product and are risk-neutral. The assumption of homogeneous consumer valuation enables one to see the incentive and the ability of the high-quality firm to signal its quality under the most intense price competition. The assumption of risk-neutral consumers enables one to focus on the role of signaling played by money-back guarantees, instead of the role of insurance. Thus, throughout this model, I shall use a representative consumer to represent all consumers. A consumer receives a value r from a product which does not break and receives 0 if the product breaks or if he does not buy any of these two goods. He has unit demand for the product, meaning that he decides if buying a unit of good 1, a unit of good 2, or not buying at all. The goods are experience goods, meaning that he does not know the qualities of the goods until he consumes one of them. Hence, the consumer's utility in different states is

⁶ R_q cannot be greater than 1. If so, the moral hazard problem on the consumer side arises. That is, the consumer is able to arbitrage by buying the good, breaking it, and returning it immediately.

⁷ There is a technical difficulty in relaxing the refund strategy as a continuous policy. Hence, the current essay only considers the two extreme policies.

$$U(\Gamma_q) = \begin{cases} r - P_q & \text{if he buys the good } q \text{ and it works} \\ 0 - P_q + R_q P_q & \text{if he buys the good } q \text{ and it fails} \\ 0 & \text{if he does not buy the good.} \end{cases}$$

Initially, nature assigns symmetrically whether firm 1 produces the high-quality good and firm 2 produces the low-quality good or the reverse. Thus, before observing the two contracts, *ex ante*, the consumer's beliefs over the qualities of the two products are the same. That is, he believes that each product is of high quality with probability 1/2 and of low quality with probability 1/2. Denote as $\mu^0 \equiv 1/2$ the consumer's prior beliefs. After observing the two contracts, *ex post*, the consumer will update his beliefs over the qualities of the two goods. Denote as $\mu(\Gamma_q, \Gamma_s)$ the updated belief over the event that firm selling contract Γ_q is the high quality firm, given the contract Γ_s sold by the other firm. Then the consumer's expected utility with contract Γ_q over the updated belief $\mu(\Gamma_q, \Gamma_s)$ is

$$\mu[\pi_H(r - P_q) + (1 - \pi_H)(R_q P_q - P_q)] + (1 - \mu)[\pi_L(r - P_q) + (1 - \pi_L)(R_q P_q - P_q)].$$

Define $Q(\mu(\cdot)) \equiv [\mu\pi_H + (1 - \mu)\pi_L]$ as the expected quality over the updated belief $\mu(\cdot)$, then the consumer's expected utility is

$$EU(\Gamma_q, \mu) = \begin{cases} Q(\mu)(r - P_q) + [1 - Q(\mu)](R_q P_q - P_q) & \text{if he buys } \Gamma_q \\ 0 & \text{if he does not buy it.} \end{cases}$$

The fact that $\mu(\cdot)$ may not be 1 when firm q is really of high quality reflects the lack of information of product qualities for the consumer. Note that since the consumer knows one of the products is of high quality and the other of low quality, he should update his beliefs about the two product qualities simultaneously. That is, I require $\mu(\Gamma_1, \Gamma_2) + \mu(\Gamma_2, \Gamma_1) = 1 \forall \Gamma_1$ and Γ_2 , as in Hertzendorf and Overgaard (1998). This requirement makes sense for a rational consumer, who knows that there is one product of each type in

the market and must decide whether to buy a unit of good 1, a unit of good 2, or not to buy at all. Moreover, it contrasts with the monopoly case in which a monopolist sells a product with one of the two types of qualities. In the monopoly case, since there is only one type of product in the market and consumers' decisions are simply buying the good or not, it is not necessary to make this requirement. In fact, it is not a requirement for the out-of-equilibrium belief in the notion of the sequential equilibrium introduced by Kreps and Wilson (1982).

The cost structures of the two firms are described as follows. Denote as C_H and C_L the constant unit costs to produce the high and low quality goods, respectively. It costs more to produce the high quality good, which means $C_H > C_L$. Yet, the unit cost of quality is lower for the high-quality good, which means $C_H/\pi_H < C_L/\pi_L$. This assumption is crucial for firm H being able to separate from firm L by giving the consumer a full refund. However, we will see in the analysis that, in equilibrium firm H may not give a full refund, and even if it does, it may not charge a different price from firm L. In order to have a nonnegative profit, the firm producing quality π_q must set its price at least as high as C_q if it offers no refund and at least as high as C_q/π_q if it offers a full refund. Thus, these two conditions imply that the firm producing low quality has a cost advantage if neither firm offers refunds, whereas the firm producing high quality has a cost advantage if both firms offer refunds. Moreover, the social benefit of each good, which is the consumer's valuation minus the cost of the good, is greater than 0, i.e., $C_H < \pi_H r$ and $C_L < \pi_L r$. However, if the consumer can correctly identify product qualities when each firm prices at its unit cost, he will choose the high-quality good rather than the other one, i.e., $\pi_H r - C_H > \pi_L r - C_L$. It also implies that the cost differential between the two

products is less than the differential of the consumer's expected valuation for these two products.

The time line for the four-stage game is depicted in Figure 3.1. Initially, nature assigns symmetrically whether firm 1 produces the high-quality good and firm 2 produces the low-quality good or the reverse. Denote the firm producing the high-quality good as firm H, and the other firm as firm L. Each firm knows its own quality and its unit cost of production and hence those of its competitor. However, the consumer does not observe this move by nature. In the second stage, each firm simultaneously chooses a refund policy R_q . In the third stage, after observing the refund policy of its competitor, each firm simultaneously chooses its price P_q .⁸ Finally, in the fourth stage, after observing the two contracts $\Gamma=(\Gamma_H, \Gamma_L)$ but not the quality or the cost of each good, the consumer makes his purchase decision according to his updated beliefs about product qualities to maximize his expected utility. It follows that the normalized demand for each good is

$$D_q(\Gamma_q, \Gamma_s, \mu(.)) = \begin{cases} 1 & \text{if } EU_q > EU_s, \text{ and } EU_q \geq 0 \\ \frac{1}{2} & \text{if } EU_q = EU_s \geq 0 \\ 0 & \text{if } EU_q < \text{Max}\{EU_s, 0\}, \end{cases}$$

where $q, s \in \{H, L\}$, $s \neq q$.⁹

⁸ The consideration of refund strategy and price strategy in different stages can be seen in Shieh (1996).

⁹ The demand can take any number between 0 and 1 when expected utilities of two contracts are the same. To simplify the model, it is assumed to be 1/2. This should not affect the results too much. Moreover, I assume that if consumers are indifferent between buying and not buying, they will buy the good.

Firms play Bertrand-Nash strategies simultaneously to maximize their own profits in the second and third stage. In the third stage, each firm chooses its own price given its own and its rival's refund policies and consumer beliefs. That is,

$$P_q \in \operatorname{argmax}_{P_q} \Pi_q = [P_q - C_q - (1 - \pi_q)R_q P_q] D_q(\Gamma_q, \Gamma_s, \mu(.)),$$

where C_q is the constant marginal cost and $(1 - \pi_q)R_q P_q$ is the expected amount of refunds.

The optimal P_q is a function of $\mu(\Gamma_q, \Gamma_s)$, Γ_s , R_q , C_q , and C_s . In the second stage, each firm chooses its own refund policy given its own and its rival's pricing strategies and consumers beliefs. That is,

$$R_q \in \operatorname{argmax}_{R_q \in \{0,1\}} \Pi_q = [P_q(.) - C_q - (1 - \pi_q)R_q(.,.,.)P_q(.)] D_q(\Gamma_q, \Gamma_s, \mu(.)).$$

Note that in order to have nonnegative profits, each firm has to set up its contract so that $[P_q - C_q - (1 - \pi_q)P_q R_q]$ and D_q are nonnegative.¹⁰ In particular, $P_q \geq C_q / [1 - (1 - \pi_q)R_q]$ must hold. For example, in the case of $(R_H, R_L) = (0, 0)$, I shall consider the prices only in the area of $(P_H, P_L) \in [C_H, \pi_H P] \times [C_L, \pi_L P]$.¹¹

To obtain the equilibria for this game, I proceed by backward induction. Since the main feature of the model is multiple senders and one receiver, it is not appropriate to immediately apply the notion of sequential equilibrium, suggested by Kreps and Wilson (1982). Nor is it appropriate to apply directly the formal refinements of equilibrium used in the standard one-sender-and-one-receiver signaling game, such as the Intuitive Criterion by Cho and Kreps (1987) and the Criterion of Universal Divinity by Banks and

¹⁰ These conditions can be viewed as Individual Rationality conditions for firms and the consumer.

¹¹ The pricing strategy space for firm L extends to $\pi_H P$ due to information asymmetry to the consumer. In the full information case, it can only be as high as $\pi_L P$.

Sobel (1987). Thus, in the spirit of equilibrium notions for a standard signaling game, I shall first provide a definition of sequential equilibrium for this game, then propose selection procedures to delete nonsensible equilibria. Specifically, in order to get a unique refined equilibrium in the subgames of no-refund and of refund-refund, I adopt the Pareto-Perfection argument, in which the equilibrium is picked up by both firms to maximize their own profits among those multiple equilibria.

To analyze whether prices themselves can costlessly signal product qualities, I first find the optimal price for each firm when the consumer can observe the quality of each good. Next, I find the optimal price for each firm when the consumer cannot observe the quality of each good but knows there is exactly one good for each quality. Then, I compare the optimal price pairs under these two structures to see if prices themselves can costlessly signal product qualities.

3.2.2. The Complete Information Structure

This section analyzes the complete information equilibrium for each firm and the consumer. The results in this information structure become the benchmarks to be compared with outcomes under incomplete information.

Proceeding by backward induction, I first find the optimal price chosen by each firm for each possible pair of refund choices. Then, the optimal refund policy is determined for each firm.

There are four possible pairs of refund choices, namely, $(R_H, R_L)=(0,0)$, $(0,1)$, $(1,0)$, or $(1,1)$. With informed consumers, the two firms play a Bertrand game, due to the homogeneity of consumers. In the case of $(R_H, R_L)=(0,0)$, since the consumer can correctly identify product qualities, i.e., $\mu((P_H, 0), (P_L, 0))=1$ and $\mu((P_L, 0), (P_H, 0))=0$,

his decision of buying from firm H, firm L, or not buying depends on which of $\pi_H r - P_H$, $\pi_L r - P_L$, or 0 is largest. Firm H gets the entire market when $\pi_H r - P_H > \max\{0, \pi_L r - P_L\}$ or $\pi_H r - P_H \geq 0 > \pi_L r - P_L$. In (P_H, P_L) space, Figure 3.2. illustrates the regions where firm H gets the entire market, where firm L gets the entire market, and where both firms share the market evenly. It shows that both firms compete with each other along the line $\pi_H r - P_H = \pi_L r - P_L$, denoted as *the competing line*. The equilibrium yields when the price level of one firm reaches its lower bound, which is its marginal cost, and the other firm seizes the entire market by lowering its price level just below the price defined by the competing line. From the assumption of $C_H - C_L < (\pi_H - \pi_L)r$, it turns out that firm L charges C_L and firm H gets the entire market by charging $C_L + (\pi_H - \pi_L)r - \varepsilon$ with a sufficiently small ε , which is reported in the following lemma.

Lemma 3.1. *When product quality is observable to the consumer before purchases and neither firm offers a refund, the equilibrium prices, demands, and profits for each firm are as follows: $P_H^* = C_L + (\pi_H - \pi_L)r - \varepsilon$, $P_L^* = C_L$, $D_H^* = 1$, $D_L^* = 0$, $\Pi_H^* = -C_H + C_L + (\pi_H - \pi_L)r - \varepsilon$, $\Pi_L^* = 0$. The consumer's expected utility is $EU^* = \pi_L r - C_L + \varepsilon$.*

All of the proofs are contained in Appendix B. Using the same argument, I find the equilibrium for the other three pairs of refund choices. Then, I obtain the optimal contract for each firm when the consumer observes product qualities before purchases, which leads to Proposition 3.1.

Proposition 3.1. *The four equilibria in the complete information are as follows.*

- (1) $\Gamma_H^* = (C_L + (\pi_H - \pi_L)r - \varepsilon, 0)$, $\Gamma_L^* = (C_L, 0)$, $D_H^* = 1$, and $D_L^* = 0$.
- (2) $\Gamma_H^* = ([C_L + (\pi_H - \pi_L)r] / \pi_H - \varepsilon, 1)$, $\Gamma_L^* = (C_L / \pi_L, 1)$, $D_H^* = 1$, and $D_L^* = 0$.

(3) $\Gamma_H^* = (C_L + (\pi_H - \pi_L)r - \varepsilon, 0)$, $\Gamma_L^* = (C_L/\pi_L, 1)$, $D_H^* = 1$, and $D_L^* = 0$.

(4) $\Gamma_H^* = ((C_L + (\pi_H - \pi_L)r)/\pi_H - \varepsilon, 1)$, $\Gamma_L^* = (C_L, 0)$, $D_H^* = 1$, and $D_L^* = 0$.

Overall, with a sufficiently small difference, the consumer obtains expected utility

$EU^* = \pi_L r - C_L$, the profit for each firm is $\Pi_H^* = -C_H + C_L + (\pi_H - \pi_L)r$ and $\Pi_L^* = 0$.

Proposition 3.1 points out that each firm is indifferent between offering a refund and not offering. Specifically, the optimal strategy for each firm is (P_H^*, R_H^*) such that $[1 - (1 - \pi_H)R_H^*]P_H^* = C_L + (\pi_H - \pi_L)r - \varepsilon$ and (P_L^*, R_L^*) such that $[1 - (1 - \pi_L)R_L^*]P_L^* = C_L$. Since this essay only considers the signaling role of the refund policy, with informed consumers, it is clear that neither firm needs to signal its quality through refunds. Hence, each firm is indifferent between offering a refund and not offering as long as the prices and the refund fractions satisfy the above respective conditions. In particular, it shows that high price corresponds to offering refunds and vice versa. Furthermore, since the quality advantage outweighs the cost disadvantage for firm H in the price competition, firm H gets the entire market and makes a profit no matter what refund policy is chosen by each firm.

3.2.3. The Incomplete Information Structure

This section characterizes the equilibria with incomplete information on the consumer side and discusses the effect of incomplete information on firms and the consumer. Before proceeding, I first introduce the basic notion of equilibrium for this game, and examine the issue of whether prices can costlessly signal product qualities with such information asymmetry.

3.2.3.1. Sequential equilibrium

To simplify the analysis, throughout the essay I shall restrict attention to pure strategy equilibrium. Following Hertzendorf and Overgaard (1998a and 1998b), the basic notion of equilibrium is defined as follows.

Definition 3.1. *A Sequential Equilibrium for this game is a pair of price-refund strategies $(\hat{P}_H(\hat{R}_H), \hat{R}_H)$ and $(\hat{P}_L(\hat{R}_L), \hat{R}_L)$ and a system of consumer beliefs $\mu((P_H, R_H), (P_L, R_L))$ such that:*

- (1) $(\hat{P}_H(\cdot), \hat{R}_H) \in \operatorname{argmax} \Pi_H(P_H(\cdot), R_H, \hat{P}_L, \hat{R}_L, \mu((P_H(\cdot), R_H), (\hat{P}_L, \hat{R}_L)))$ with $P_H(\cdot) \in [0, r]$ and $R_H \in \{0, 1\}$.
- (2) $(\hat{P}_L(\cdot), \hat{R}_L) \in \operatorname{argmax} \Pi_L(P_L(\cdot), R_L, \hat{P}_H, \hat{R}_H, \mu((P_L(\cdot), R_L), (\hat{P}_H, \hat{R}_H)))$ with $P_L(\cdot) \in [0, r]$ and $R_L \in \{0, 1\}$.
- (3) if $(\hat{P}_H, \hat{R}_H) \neq (\hat{P}_L, \hat{R}_L)$, then $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)) = 1$.
- (4) if $(\hat{P}_H, \hat{R}_H) = (\hat{P}_L, \hat{R}_L)$, then $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)) = 1/2$.
- (5) $\mu((P_H, R_H), (P_L, R_L)) + \mu((P_L, R_L), (P_H, R_H)) = 1$. Further, if $(P_H, R_H) = (P_L, R_L)$, then $\mu((P_H, R_H), (P_L, R_L)) = 1/2$.
- (6) Consumers believe that each firm will not set price such that $[1 - (1 - \pi_q)R_q]P_q < C_q$ since that would lose money.

Condition (1) simply requires that the high-quality firm select a strategy to maximize its profits, taking as given the strategy of the low-quality firm and the belief system of the consumer, i.e., sequential rationality. Similarly, condition (2) requires that the low-quality firm select a strategy to maximize its profits, taking as given the strategy of the high-quality firm and the belief system of the consumer. Conditions (3) and (4)

require that the belief of the consumer on the equilibrium path be consistent, given the strategies of the firms and the overall structure of the game. Condition (5) requires that the beliefs about the two products off the equilibrium path sum to one, which makes sense for rational consumers. Moreover, it requires that consumers revert to their prior beliefs if the two firms select the same strategy. This is because consumers still cannot infer information about qualities from two identical contracts. Condition (6) requires that consumers believe in the intention of firms to earn nonnegative profits.

3.2.3.2. Can prices alone signal product qualities?

In fact, when the strategy space is restricted to no refund, prices cannot signal the quality of good H. More specifically, firm L can always pool with firm H so that the consumer cannot distinguish between their goods.

Proposition 3.2. *When product quality is unobservable to the consumer before purchases, if the strategy space of two firms is restricted to no refund, the firm producing high-quality goods cannot signal its quality. That is, no separating equilibrium can be supported by any system of consumer beliefs.*

The intuition for Proposition 3.2 is as follows. High quality gives more value to the consumer than low quality does. Thus, firm L has an incentive to mimic the high-quality firm so that it can sell at a higher price. The only way for firm L to do so is to pool with firm H. Since firm H has a higher cost than firm L does, the price firm H chooses can always be mimicked by firm L. Thus, no beliefs support a separating equilibrium. The result of Proposition 3.2 also implies that the complete information equilibria cannot be sustained under incomplete information.

To support a price as a pooling equilibrium, the consumer must have appropriate out-of-equilibrium beliefs. For an upward deviating price, $1/2$ of the probability as of

high quality suffices to deter both firms from deviating upward. For a downward deviating price, the out-of-equilibrium belief is a function of the equilibrium price. That is, the size of the equilibrium price determines the range of the out-of-equilibrium belief by which the pooling equilibrium price is supported. It is harder to support a higher equilibrium price with an out-of-equilibrium belief of high probability as of high quality, so the range of plausible out-of-equilibrium beliefs is smaller and vice versa. Figure 3.3. illustrates the range of plausible out-of-equilibrium beliefs supporting the multiple pooling equilibria as in the shaded triangle area and in the black segment.

To reduce the set of multiple pooling sequential equilibria, I adopt the heuristic argument used in Hertzendorf and Overgaard (1998a), called the Pareto-Perfection argument.¹² It suggests that firms pick the strategy that is preferred by both firms, which is as if firms can coordinate implicitly *ex ante*. Obviously, a pooling equilibrium survives the Pareto-Perfection argument if there is a strong commonality of preferences across firms, which is the case for each subgame of refund choices. The unique refined pooling equilibrium is reported in the following proposition.

Proposition 3.3. *When product quality is unobservable to the consumer before purchases, if the strategy space of the two firms is restricted to no refund, there exist multiple pooling sequential equilibria. The unique refined pooling equilibrium that survives the Pareto-Perfection argument is for both firms to charge $\min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\}$ and the consumer's out-of-equilibrium belief is "whoever deviates upward is of high quality with probability 1/2 and whoever deviates downwards is of low quality".*

¹² See, for example, Bagwell and Ramey (1988), and more indirectly, Mailath et al. (1993) and Overgaard (1993). It is similar to the focal point suggested by Kreps (1990, p.415).

Knowing that firm L always mimics its behavior, firm H will choose its price as high as possible, which is $\min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\}$, so as to cover part of the loss in profits due to the loss of half of the market demand to firm L. The out-of-equilibrium belief to support it as a pooling equilibrium is that whoever deviates downward is firm L. With such a belief, the consumer will not buy at the deviating price. Firm H has no incentive to deviate, which would lead consumers to believe it to be firm L. Neither does firm L have an incentive to deviate which leads the consumer to identify it correctly. Therefore, it is a pooling equilibrium which gives both firms the highest profits.

A point worthwhile to mention is that there can be implicit collusion between the two firms. By definition, implicit collusion is a situation when the two firms compete with each other but end up with a result the same as with explicit collusion. In this model, if the two firms both produce and collude with each other, they will grab all of the consumer surplus and charge $(\pi_H + \pi_L)r/2$, which happens to be the same result as above if $2(\pi_H - \pi_L)r + C_L \geq (\pi_H + \pi_L)r/2$. Interestingly, incomplete information of product quality on consumer side provides a possible rationale of implicit collusion when each firm only uses price as its strategy.

Comparing the results under incomplete information with those under complete information, first, the consumers' expected utility decreases. Second, firm L not only can survive in the market but also gets higher positive profits than firm H does. This is due to its ability to pool with firm H. Third, profits of firm H decrease. Therefore, firm L is the biggest winner. Since the profits of firm H are lower with incomplete information than with complete information, as in Shieh (1996), firm H has an incentive to separate from firm L through other strategy such as advertising, best-price guarantees or money-back

guarantees. This essay focuses on the use of money-back guarantees to signal product quality.

3.2.3.3. Refund policies and prices as a signal of product qualities

The Equilibrium Outcomes in Stage 3. With the result that firm H cannot signal its quality by price alone, I now examine whether refunds are able to reveal product quality and whether firms will use them.

First, consider the case when only one firm offers a refund. One may wonder whether, in equilibrium, the consumer can then identify each firm's product quality according to each firm's refund choice so that the equilibrium in the price competition stage will be the same as with complete information. Suppose this is true. However, since there is uncertainty regarding which firm offers a refund, firm L offering a refund can still fool the consumer by charging at a price where firm H offering a refund would have charged. Similarly, firm L offering no refund can fool the consumer by charging at a price where firm H offering no refund would have charged. That is, when only one firm offers a refund, the complete-information equilibrium price set still cannot be sustained under incomplete information because the incentive compatibility (IC) condition is not satisfied. The following lemma summarizes this result.

Lemma 3.2. *When there is only one firm offering a refund, the two complete information equilibria together cannot be sustained under incomplete information.*

Thus, the supposition is wrong. In fact, the equilibrium price of good H when firm H offers a refund is forced downwards to a price just a little bit lower than C_L/π_L , a price which firm L cannot mimic. These results are summarized in Lemma 3.3.

Lemma 3.3. *When product quality is unobservable to the consumer before purchases*

and only one firm offers a refund, then:

(1) If it is firm H offering a refund, the equilibrium price, demand, and profits for each firm are as follows. $P_H = C_L/\pi_L - \varepsilon$, $D_H = 1$, $\Pi_H = \pi_H[C_L/\pi_L - \varepsilon - C_H/\pi_H]$, $P_L = C_L$, $D_L = 0$, $\Pi_L = 0$.

The consumer's expected utility is $EU = \pi_H(r - C_L/\pi_L + \varepsilon)$.

(2) If it is firm L offering a refund, the equilibrium is the same as with complete information. That is, the equilibrium price, demand, and profits for each firm are as follows. $P_H = C_L + (\pi_H - \pi_L)r - \varepsilon$, $D_H = 1$, $\Pi_H = -C_H + C_L + (\pi_H - \pi_L)r - \varepsilon$, $P_L = C_L/\pi_L$, $D_L = 0$, $\Pi_L = 0$.

The consumer's expected utility is $EU = \pi_L r - C_L + \varepsilon$.

When only firm H offers a refund and charges $C_L/\pi_L - \varepsilon$, the consumer gets a strong signal that the good with the price $C_L/\pi_L - \varepsilon$ is of high quality. Thus, even if firm L mimics firm H's equilibrium price in the other case, it cannot fool the consumer. Hence, firm L has no incentive to deviate. On the other hand, when only firm L offers a refund, firm L cannot mimic firm H's equilibrium price in the former case because of cost disadvantage. Hence, firm L has no incentive to deviate in this case either. Therefore, these two price sets are the equilibrium price sets with incomplete information.

Compare the outcomes in Lemma 3.3 to those under complete information. As stated in Lemma 3.3 (2), the equilibrium profits of each firm and the consumer's expected utility are the same under these two different information structures when only firm L offers a refund. However, as stated in Lemma 3.3 (1), when only firm H offers a refund, firm H's equilibrium price and profits are lower under incomplete information than those under complete information. This downward distortion comes from incomplete information of product quality on the consumer side and the incentive compatibility condition. When only firm H offers a refund, the consumer's expected

utility is higher with incomplete information than with complete information and firm L's equilibrium profit remains the same under these two information structures.

Next, consider the case when both firms offer refunds. Since $C_H/\pi_H < C_L/\pi_L$, firm H has cost advantage in offering a refund. When firm H offers a refund, firm L fails to follow this time due to cost disadvantage. Thus, by offering a refund firm H can prevent firm L from charging the same price as firm H does. Therefore, the consumer can identify product qualities according to their different prices. However, depending on the parameters, firm H may find its best interest to pool with firm L. In this case, then no separating equilibrium is sustained. This equilibrium outcome is summarized below.

Lemma 3.4. *When product quality is unobservable to the consumer before purchases and both firms offer refunds, there exists a unique separating equilibrium if $\beta \equiv (2\pi_H - \pi_L)(C_L/\pi_L) - C_H - (\pi_H - \pi_L)r > 0$. The equilibrium price, demand, and profits for each firm are as follows. $P_H = C_L/\pi_L - \varepsilon$, $D_H = 1$, $\Pi_H = \pi_H[C_L/\pi_L - C_H/\pi_H - \varepsilon]$, $P_L \in [C_L/\pi_L, (2C_L/\pi_L - C_H/\pi_H))$, $D_L = 0$, $\Pi_L = 0$. The consumer's expected utility is $EU = (\pi_H r - C_L + \varepsilon)\pi_H/\pi_L$.*

From Lemma 3.3 and Lemma 3.4, it follows that refunds together with prices can signal the true quality of the goods with a signaling cost under certain condition. Using refunds, firm H regains the whole market. But it does not receive the complete-information profit. This result is different from that of Shieh (1996) with a monopoly provider. In her paper, money-back guarantee is a costless signal to product quality. Moreover, the equilibrium prices in her paper are different from my results either. In her model, in addition to offering refunds, the high-quality type monopoly needs to raise its price to signal quality and to avoid the low-quality type monopoly from mimicking it. With a duopoly, the high-quality firm's price is raised only to reflect the cost of refunds

and is not as high as that under complete information.

The following lemma identifies the existence of multiple pooling equilibria under certain condition, and then obtains the unique refined pooling equilibrium if the Pareto-Perfection argument is accepted.¹³

Lemma 3.5. *When product quality is unobservable to the consumer before purchases and both firms offer refunds, there exist multiple pooling sequential equilibria if $\beta < 0$. If the Pareto-Perfection argument is accepted, the unique refined pooling equilibrium price*

$$\text{is } \frac{2(\pi_H - \pi_L)r + \pi_L \frac{C_H}{\pi_H}}{2\pi_H - \pi_L} \equiv \theta.$$

Lemma 3.5 illustrates that, if $\beta < 0$ holds and the Pareto-Perfection argument is accepted, firm H finds its best interest to pool with firm L at a high price, instead of separating from firm L by charging a low price. In this case, compared to the complete information refund-refund equilibrium, the pooling equilibrium price is higher than the price charged by firm H under complete information. However, the profits of firm H under incomplete information is still lower than those under complete information, due to its sharing half of market demand with firm L. Moreover, the consumer's expected utility is lower under incomplete information than under complete information, which contrasts with the separating case if $\beta > 0$.

Compared to the pooling case where neither firm offers a refund, the profits of firm H under refund-refund case is lower if $\min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\} = 2(\pi_H -$

¹³ The shaded area in Figure 3.4. shows the relation between the out-of-equilibrium beliefs and the pooling equilibrium price where neither firm has an incentive to deviate to a price higher than or equal to C_L/π_L . After considering the possible deviation to a price lower than C_L/π_L , the area for pooling equilibria may be smaller depending on parameters. See the proof of Lemma 3.5 in Appendix B for more detail.

$\pi_L)r + C_L$, and is ambiguous if $\min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\} = (\pi_H + \pi_L)r/2$. Nonetheless, the consumer's expected utility is unanimously higher under refund-refund case than under no-refund case due to the functioning of refunds.

The Equilibrium Outcomes in Stage 2. Finally, I examine the issue of whether offering a refund is in the best interest of each firm. Table 3.1. summarizes the equilibrium under alternative combinations of refund strategies. Four cases arises depending on the signs of $\{2(\pi_H - \pi_L)r + C_L - (\pi_H + \pi_L)r/2\} = \alpha$ and $\{(2\pi_H - \pi_L)(C_L/\pi_L) - C_H - (\pi_H - \pi_L)r\} = \beta$, where the former bracket determines the pooling equilibrium price in the no-refund case and the latter bracket determines either the separating equilibrium or the pooling equilibrium in the refund-refund case.

Proposition 3.4. *When product quality is unobservable to the consumer before purchases and each firm can use both price strategy and refund strategy, if $\alpha > 0$ and $\beta > 0$, then the equilibrium refund policy for each firm is $(\hat{R}_H, \hat{R}_L) = (0, 0)$ if $\eta > 0$ and*

$$(\hat{R}_H, \hat{R}_L) = (1, 0) \text{ if } \eta < 0 \text{ where } \eta = \frac{(\pi_H + \pi_L)r}{4} - \frac{C_H}{2} - \pi_H \left(\frac{C_L}{\pi_L} - \frac{C_H}{\pi_H} \right).$$

Proposition 3.5. *When product quality is unobservable to the consumer before purchases and each firm can use both price strategy and refund strategy, if $\alpha > 0$ and $\beta < 0$, then the equilibrium refund policy for each firm is $(\hat{R}_H, \hat{R}_L) = (0, 0)$ if $\eta > 0$ and (1, 1).*

Proposition 3.6. *When product quality is unobservable to the consumer before purchases and each firm can use both price strategy and refund strategy, if $\alpha < 0$ and $\beta > 0$, then the equilibrium refund policy for each firm is $(\hat{R}_H, \hat{R}_L) = (0, 0)$.*

Proposition 3.7. *When product quality is unobservable to the consumer before*

purchases and each firm can use both price strategy and refund strategy, if $\alpha < 0$ and $\beta < 0$, then the equilibrium refund policy for each firm is $(\hat{R}_H, \hat{R}_L) = (0, 0)$.¹⁴

Table 3.2. summarizes the equilibrium refund strategies for alternative combinations of parameters, according to Proposition 3.4 to Proposition 3.7. It follows that $(\hat{R}_H, \hat{R}_L)$ may be (0, 0), (1, 1), or (1, 0), depending on the parameters. But, $(\hat{R}_H, \hat{R}_L) = (0, 1)$ is never an equilibrium set. Intuitively, firm L has no incentive to reveal its quality type. Thus, it will not offer a refund if firm H offers no refund. On the other hand, it will offer a refund if it is certain that firm H prefers offering refunds and pooling in price.

Propositions 3.4, 3.5, 3.6, and 3.7 also show the possibility of staying in the market for firm L. This contrasts with the complete information case, where it is impossible for firm L to stay in the market, because of zero demand. With incomplete information, firm L can pool with firm H and get half of market demand under certain conditions.

If we focus on the separating equilibrium as in Shieh (1996), the only separating equilibrium in the duopoly with incomplete information is $(\hat{R}_H, \hat{R}_L) = (1, 0)$ existing under certain conditions of parameters. This contrasts with the result in Shieh (1996) with monopoly, where both (1, 1) and (1, 0) are separating equilibria. Moreover, Shieh (1996) concludes that the money-back guarantee itself cannot signal product quality, because in equilibrium the L-type firm can also offer refunds. This differs from my result, where the

¹⁴ Although the case of $(\hat{R}_H, \hat{R}_L) = (1, 1)$ is also a Nash equilibrium, it is dominated by (0, 0) for both firms and thus is eliminated.

money-back guarantee itself can signal product qualities in duopoly, because in the separating equilibrium firm H offers refunds and firm L does not. Finally, in Shieh (1996), the provision of a money-back guarantee allows the H-type firm to signal with complete information price, thus there is no signaling cost in monopoly. This also differs from my result, where in duopoly the price of firm H is distorted downwards and the profits of firm H are lower than with complete information. These differences show that it is inappropriate to directly apply the results in monopoly to duopoly.

Table 3.3. summarizes the equilibrium results under alternative information structures, strategy spaces, and combinations of parameters. With incomplete information, the consumer may be better off if firms can choose to offer refunds than if firms cannot. Specifically, compared to the equilibrium results with complete information, the expected utility of the consumer is lower in the cases of $(\hat{R}_H, \hat{R}_L) = (1, 1)$ and $(0, 0)$, and it is higher in the case of $(1, 0)$. Since $(\hat{R}_H, \hat{R}_L) = (1, 1)$ is an equilibrium set only when both firms pool together, it hurts the consumer because of the high pooling price. It also yields a pooling equilibrium for the case of $(\hat{R}_H, \hat{R}_L) = (0, 0)$, thus it hurts the consumer as well. When only firm H offers refunds in equilibrium, consumers benefit from lacking information about product qualities. The reason is clear. With complete information, firm H can get the entire market without decreasing its price. However, with incomplete information, firm H must decrease its price low enough to convince the consumer about its quality. Thus, the consumer benefits from lacking information. This result is not rare. For example, the second firm gets a lower profit in the Stackelberg model, in which it observes the first firm's strategy, than in the Cournot model, in which it does not observe the other firm's strategy.

Define social welfare as the sum of the profits of both firms and the expected utility of the consumer, I obtain the following results when comparing the two information structures.

Lemma 3.6. *Compared to the complete information case, social welfare is the same under incomplete information if $(\hat{R}_H, \hat{R}_L) = (1, 0)$ and lower under incomplete information if $(\hat{R}_H, \hat{R}_L) = (0, 0)$ or $(1, 1)$.*

Lemma 3.6 points out that social welfare with incomplete information regains to the complete information level only if firm H offers refunds and firm L does not. Since $(\hat{R}_H, \hat{R}_L) = (1, 0)$ is an equilibrium only when $\alpha > 0$, $\beta > 0$, and $\eta < 0$, for those parameters not satisfying these three conditions, incomplete information on the consumer side hurts the society as a whole. Take another point of view that $(\hat{R}_H, \hat{R}_L) = (1, 0)$ corresponds to a separating equilibrium and $(\hat{R}_H, \hat{R}_L) = (0, 0)$ or $(1, 1)$ corresponds to pooling equilibria, a rule of thumb is that consumers are better off and the society is not hurt if in equilibrium only firm H offers refunds.

3.3. Heterogeneous Consumers, Same Unit Cost, and Same Timing of Firm Decisions

This section analyzes the second case, where consumers have different valuations on a working product and each firm with the same unit costs of production simultaneously decides a refund policy and a price.

3.3.1. The Basic Model

The model in this case is similar to that in the homogeneous-consumer case. Thus, I omit the same parts and only describe the different features.

Consumers are heterogeneous in their monetary valuation of a working product and are risk-neutral, but are homogeneous in the belief about the quality of a product. Let r represent a potential consumer's valuation of a working product, which serves as an index. These valuations correspond to an interval $[0,1]$ with a uniform distribution and total mass 1.¹⁵

When consumers can correctly identify product qualities, i.e., the case of complete information and the case of separating equilibria of incomplete information, it is easy to determine the demand facing each firm for a given profile of prices and refund choices. Define as $r^* \equiv \frac{[1-(1-\pi_H)R_H]P_H - [1-(1-\pi_L)R_L]P_L}{\pi_H - \pi_L}$ the valuation of the marginal consumer who is indifferent between buying a unit of good H and a unit of good L; define as $r^{**} \equiv \frac{[1-(1-\pi_L)R_L]P_L}{\pi_L}$ the valuation of the marginal consumer who is indifferent between buying a unit of good L and not buying; and define as

$r^{***} \equiv \frac{[1-(1-\pi_H)R_H]P_H}{\pi_H}$ the valuation of the marginal consumer who is indifferent

between buying a unit of good H and not buying. Hertzendorf and Overgaard (1998a) show that in any Nash equilibrium of the complete information game $0 < \theta^{**} < \theta^* < 1$, where $\theta^* = (P_H - P_L) / (\pi_H R_H - \pi_L R_L)$ and $\theta^{**} = P_L / \pi_L$. Thus, by treating $[1-(1-\pi_H)R_H]P_H$ and $[1-(1-\pi_L)R_L]P_L$ in my model as P_H and P_L in their model, I can directly apply their result and obtain that $0 < r^{***} < r^* < 1$ in any Nash equilibrium of the complete information.

¹⁵ The uniform distribution is convenient because it generates linear demand. This assumption can be seen in many papers, such as in Sheih (1993) and Hertzendorf and Overgaard (1998).

Consequently, the demands $D_H(\Gamma_H, \Gamma_L, 1, \pi_H, \pi_L)$ and $D_L(\Gamma_L, \Gamma_H, 0, \pi_H, \pi_L)$ in the case of complete information and in the case of separating equilibria of incomplete information are as follows:

$$D_H = 1 - r^* = 1 - \frac{[1 - (1 - \pi_H)R_H]P_H - [1 - (1 - \pi_L)R_L]P_L}{\pi_H - \pi_L},$$

$$D_L = r^* - r^{**} = \frac{[1 - (1 - \pi_H)R_H]P_H - [1 - (1 - \pi_L)R_L]P_L}{\pi_H - \pi_L} - \frac{[1 - (1 - \pi_L)R_L]P_L}{\pi_L},$$

where the third argument in function $D_q(\cdot)$ is the belief of consumers about the event that good q is of high quality, $q \in \{H, L\}$. The condition $0 < r^{**} < r^* < 1$ ensures that both firms sell goods to consumers in equilibrium. It can be checked that this is the case in the equilibria I will consider below.

In the case of pooling equilibria of incomplete information, i.e., $\Gamma_H = \Gamma_L$, consumers cannot identify product qualities and thus revert to their prior beliefs $\mu^0 = 1/2$. Hence, by assuming that both firms share total demand evenly, the demand for each firm is:

$$D_q((P, R), (P, R), \frac{1}{2}, \pi_H, \pi_L) = \frac{1}{2} \left[1 - \frac{2P}{\pi_H + \pi_L} \left(1 - \left(1 - \frac{\pi_H + \pi_L}{2} \right) R \right) \right], q \in \{H, L\}.$$

Following Hertzendorf and Overgaard (1998a and 1998b), I assume that the cost structures of producing these two types of products are the same and normalize them to zero. This implies that the cost of production is independent of quality.

The time line for the three-stage game is depicted in Figure 3.5. Initially, nature assigns symmetrically whether firm 1 produces the high-quality good and firm 2 produces the low-quality good or the reverse. Denote the firm producing the high-quality good as firm H, and the other firm as firm L. Each firm knows its own quality and hence

that of its competitor. However, the consumer does not observe this move by nature. In the second stage, each firm simultaneously chooses a price P_q and a refund policy R_q , where contract $\Gamma_q = (P_q, R_q) \in [0, 1] \times \{0, 1\}$. In the third stage, after observing the two contracts $\Gamma = (\Gamma_1, \Gamma_2)$ but not the quality of each good, each consumer makes his purchase decision according to his updated beliefs about product qualities to maximize his expected utility. I use the following notation to represent the profits of firm q :

$$\Pi_q(\Gamma_q, \Gamma_k, \mu(\Gamma_q, \Gamma_k), \pi_H, \pi_L) = [1 - (1 - q)R_q]P_q D_q(\Gamma_q, \Gamma_k, \mu(\Gamma_q, \Gamma_k), \pi_H, \pi_L),$$

where $q, k \in \{H, L\}$ and $q \neq k$. Firms play simultaneously to maximize their own profits in the second stage. That is, each firm chooses its own price and refund policy given its rival's price and refund policy and consumers' belief system.

Since the main feature of the model considered in this essay is multiple senders and one receiver, it is not appropriate to immediately apply the notion of sequential equilibrium suggested by Kreps and Wilson (1982). Nor is it appropriate to apply directly the formal refinements of equilibrium used in the standard one-sender-and-one-receiver signaling game, such as the Intuitive Criterion by Cho and Kreps (1987) and the Criterion of Universal Divinity by Banks and Sobel (1987). Thus, in the spirit of equilibrium notions for a standard signaling game, I shall first present a definition of sequential equilibrium for this game, then propose selection procedures to delete nonsensible equilibria. Specifically, following Hertzendorf and Overgaard (1998b), I use REDE (resistant to equilibrium defection) to refine separating sequential equilibria to refined separating equilibria for each of π_L such that the set of separating sequential equilibria is nonempty. Further, I employ their incentive argument to refine the set of

pooling sequential equilibria. It turns out that no pooling sequential equilibrium survives this argument.

To analyze whether prices themselves can costlessly signal product qualities, I first find the optimal price-refund strategy for each firm in the complete information, i.e., when consumers can observe the quality of each good. Next, I find the optimal price for each firm in the incomplete information. Then, I compare the optimal price pairs under these two structures to see if prices themselves can costlessly signal product qualities.

3.3.2. The Complete Information Structure

This section characterizes the complete information equilibria, which serve as the benchmarks to compare with outcomes in the incomplete information structure. To simplify the analysis, I assume π_H as 0.9.¹⁶ Moreover, keep in mind that this essay only considers two extreme refund fractions, i.e., $R_q=0$, or $R_q=1$, $q \in \{H, L\}$.

Proposition 3.8. Define $P_{H0}^* \equiv \frac{2\pi_H(\pi_H - \pi_L)}{4\pi_H - \pi_L}$, $P_{H1}^* \equiv \frac{2(\pi_H - \pi_L)}{4\pi_H - \pi_L}$,

$P_{L0}^* \equiv \frac{\pi_L(\pi_H - \pi_L)}{4\pi_H - \pi_L}$, and $P_{L1}^* \equiv \frac{(\pi_H - \pi_L)}{4\pi_H - \pi_L}$. Under complete information the equilibrium

strategies, demands, and profits are:

$(I_H^*, I_L^*) = ((P_{H0}^*, 0), (P_{L0}^*, 0)), ((P_{H1}^*, 1), (P_{L0}^*, 0)), ((P_{H0}^*, 0), (P_{L1}^*, 1)),$ and $((P_{H1}^*,$

$1), (P_{L1}^*, 1))$. $D_H^* = \frac{2\pi_H}{4\pi_H - \pi_L}$, and $D_L^* = \frac{\pi_H}{4\pi_H - \pi_L}$. $\Pi_H^* = \frac{4\pi_H^2(\pi_H - \pi_L)}{(4\pi_H - \pi_L)^2}$, and

¹⁶ Assuming π_H as 0.9, instead of 1.0 as in Hertzendorf and Overgaard (1998b), could avoid the problem of multiple equilibria for firm H due to the indifference of firm H between offering refunds and not, since no consumer will return the high-quality product if it always works.

$$\Pi_L^* = \frac{\pi_H \pi_L (\pi_H - \pi_L)}{(4\pi_H - \pi_L)^2}.$$

Proposition 3.8 points out that each firm is indifferent between offering a refund and not offering. Specifically, the optimal strategies for each firm are (P_H^*, R_H^*) such that $[1-(1-\pi_H)R_H^*]P_H^* = 2\pi_H(\pi_H-\pi_L)/(4\pi_H-\pi_L)$ and (P_L^*, R_L^*) such that $[1-(1-\pi_L)R_L^*]P_L^* = \pi_L(\pi_H-\pi_L)/(4\pi_H-\pi_L)$. Since this essay only considers the signaling role of the refund policy, with informed consumers, it is clear that neither firm needs to signal its quality through refunds. Hence, each firm is indifferent between offering a refund and not offering as long as the relation between the price and the refund fraction satisfies the above conditions. In particular, it shows that high price corresponds to offering refunds and vice versa. Moreover, the demand of firm H is two times as high as that of firm L. Finally, firm H earns higher profits than firm L.

3.3.3. The Incomplete Information Structure

This section characterizes the equilibria with incomplete information on the consumer side and discusses the effect of incomplete information on the consumer's expected utility and the demand, the pricing and refund strategies, and profits for each firm. I first introduce the basic notion of equilibrium for this game, and then examine the issue of whether the optimal price-refund strategies in the complete information can constitute a separating equilibrium in the incomplete information.

3.3.3.1. The definition of sequential equilibrium

To simplify the analysis, throughout the essay I shall restrict attention to pure strategy equilibrium. Following Hertzendorf and Overgaard (1998a, 1998b), the basic notion of equilibrium is defined as follows.

Definition 3.2. A Sequential Equilibrium for this game is a pair of price-refund

strategies $(\hat{P}_H, \hat{R}_H)$ and $(\hat{P}_L, \hat{R}_L)$ and a system of consumer beliefs $\mu((P_H, R_H), (P_L, R_L))$

such that:

(1) $(\hat{P}_H, \hat{R}_H) \in \operatorname{argmax} \Pi_H(P_H, R_H, \hat{P}_L, \hat{R}_L, \mu((P_H, R_H), (\hat{P}_L, \hat{R}_L)), \pi_H, \pi_L)$ with $P_H \in [0, r]$ and $R_H \in \{0, 1\}$.

(2) $(\hat{P}_L, \hat{R}_L) \in \operatorname{argmax} \Pi_L(P_L, R_L, \hat{P}_H, \hat{R}_H, \mu((P_L, R_L), (\hat{P}_H, \hat{R}_H)), \pi_H, \pi_L)$ with $P_L \in [0, r]$ and $R_L \in \{0, 1\}$.

(3) if $(\hat{P}_H, \hat{R}_H) \neq (\hat{P}_L, \hat{R}_L)$, then $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)) = 1$.

(4) if $(\hat{P}_H, \hat{R}_H) = (\hat{P}_L, \hat{R}_L)$, then $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)) = 1/2$.

(5) $\mu((P_H, R_H), (P_L, R_L)) + \mu((P_L, R_L), (P_H, R_H)) = 1$. Further, if $(P_H, R_H) = (P_L, R_L)$, then $\mu((P_H, R_H), (P_L, R_L)) = 1/2$.

Definition 3.2 differs from Definition 3.1 only in that the price of each firm is dependent of its refund policy in the first case, whereas it is independent in the second case. Moreover, since the unit cost is normalized to zero in the second case, Definition 3.2 need not condition (6) of Definition 3.1.

3.3.3.2. The separating sequential equilibria

Having defined the notion of sequential equilibrium for this game, I start the analysis by giving necessary conditions for the price-refund pairs to constitute a separating equilibrium profile.

Lemma 3.7. Necessary conditions for $(\hat{P}_H, \hat{R}_H) \neq (\hat{P}_L, \hat{R}_L)$ to be a separating equilibrium profile are:

(1) $\Pi_L((\hat{P}_L, \hat{R}_L), (\hat{P}_H, \hat{R}_H), 0, \pi_H, \pi_L) \geq \Pi_L((\hat{P}_H, \hat{R}_H), (\hat{P}_H, \hat{R}_H), 1/2, \pi_H, \pi_L)$

$$(2) \Pi_H((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L), 1, \pi_H, \pi_L) \geq \Pi_H((\hat{P}_L, \hat{R}_L), (\hat{P}_L, \hat{R}_L), 1/2, \pi_H, \pi_L)$$

$$(3) (\hat{P}_L, \hat{R}_L) \in \operatorname{argmax} \Pi_L((P_L, R_L), (\hat{P}_H, \hat{R}_H), 0, \pi_H, \pi_L) \text{ with } (P_L, R_L) \in [0, 1] \times \{0, 1\}.$$

The three conditions in Lemma 3.7 are almost standard. Conditions (1) and (2) require that each firm prefers the putative separating profile to mimicking the strategy of the other firm, where consumers revert to their prior beliefs if it mimics the strategy of the other firm. These two conditions are implied by conditions (1), (2), (4), and (5) of the definition of a sequential equilibrium. Finally, given a separating equilibrium in which firm L faces the worst possible consumer beliefs, i.e., $\mu=0$, firm L should pick its strategy to maximize profits. Consequently, the optimal strategy of firm L given the strategy $(\hat{P}_H, \hat{R}_H)$ of firm H is presented in the following lemma.

Lemma 3.8. *In any separating equilibrium profile $\{(\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)\}$, the best reply of firm L is either $\Gamma_L = ((\pi_L/(2\pi_H))[1 - (1 - \pi_H)\hat{R}_H] \hat{P}_H, 0)$ or $\Gamma_L = ((1/(2\pi_H))[1 - (1 - \pi_H)\hat{R}_H] \hat{P}_H, 1)$.*

Unlike in the case of the price-advertising strategy space studied by Hertzendorf and Overgaard (1998b), which results in a unique strategy for firm L, there are two price-refund separating strategies for firm L to best respond to the strategy of firm H. In fact, these two strategies are common in that $[1 - (1 - \pi_L)R_L]P_L = (\pi_L/(2\pi_H))[1 - (1 - \pi_H)\hat{R}_H] \hat{P}_H$.

Next, I examine the issue of whether either of the two complete information equilibrium profiles can constitute a separating equilibrium profile in the incomplete information case.

Proposition 3.9. *None of the complete information equilibrium profiles (Γ_H^*, Γ_L^*) can constitute a separating equilibrium in the incomplete information case.*

Proposition 3.9 points out that firm L always has an incentive to mimic the strategy of firm H if firm H adopts any of its complete information equilibrium strategies. In other words, firm H cannot separate from firm L and consumers will only observe two identical contracts under incomplete information. Thus, firm H has to deviate from its complete information equilibrium strategies so as to prevent firm L from mimicking. Since a strategy includes two choices, i.e., price and refund policy, such deviation can be made by changing price only, refund policy only, or both.

Consider condition (1) of Lemma 3.7. Substituting in for the best response of firm L derived in Lemma 3.8, I obtain:

$$\frac{\pi_L}{2\pi_H} \frac{[1 - (1 - \pi_H)\hat{R}_H]^2 \hat{P}_H}{(\pi_H - \pi_L)} \geq [1 - (1 - \pi_L)\hat{R}_H] \left[1 - \frac{2\hat{P}_H}{\pi_H + \pi_L} \left(1 - (1 - \frac{\pi_H + \pi_L}{2})\hat{R}_H \right) \right]. \quad (3.1)$$

Inequality (3.1) defines the relation between $\hat{P}_H$ and $\hat{R}_H$ so as to deter firm L from mimicking. It can be expressed as

$$\hat{P}_H \geq \bar{P}_H(\hat{R}_H; \pi_L) \equiv \frac{1}{\frac{\pi_L[1 - (1 - \pi_H)\hat{R}_H]}{\pi_H[1 - (1 - \pi_L)\hat{R}_H]} + \frac{2[1 - (1 - \frac{\pi_H + \pi_L}{2})\hat{R}_H]}{2(\pi_H - \pi_L)} + \frac{2[1 - (1 - \frac{\pi_H + \pi_L}{2})\hat{R}_H]}{(\pi_H + \pi_L)}}. \quad (3.2)$$

In particular, $\bar{P}_H(0; \pi_L) = \frac{2\pi_H(\pi_H - \pi_L)(\pi_H + \pi_L)}{\pi_L(\pi_H + \pi_L) + 4\pi_H(\pi_H - \pi_L)} = \frac{1.458 - \pi_L^2}{\pi_L^2 - 2.7\pi_L + 3.24}$; and

$$\bar{P}_H(1; \pi_L) = \frac{2(\pi_H - \pi_L)}{3\pi_H - 2\pi_L} = \frac{1.8 - 2\pi_L}{2.7 - 2\pi_L}.$$

Next consider condition (2) of Lemma 3.7, which can be written as

$$[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H \left[1 - \frac{(2\pi_H - \pi_L)[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H}{2\pi_H(\pi_H - \pi_L)} \right]$$

$$\geq [1 - (1 - \pi_H) \hat{R}_L] \frac{\hat{P}_L}{2} \left[1 - \frac{2\hat{P}_L}{\pi_H + \pi_L} \left(1 - (1 - \frac{\pi_H + \pi_L}{2}) \hat{R}_L \right) \right].$$

Substituting in for the best response of firm L derived in Lemma 3.8, I obtain:

$$\begin{aligned} & [1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H \left[1 - \frac{(2\pi_H - \pi_L)[1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H}{2\pi_H(\pi_H - \pi_L)} \right] \\ & \geq \frac{\pi_L[1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H}{4\pi_H} \left[1 - \frac{\pi_L[1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H}{\pi_H(\pi_H + \pi_L)} \right] \text{ for } \hat{R}_L = 0; \end{aligned} \quad (3.3)$$

$$\begin{aligned} & [1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H \left[1 - \frac{(2\pi_H - \pi_L)[1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H}{2\pi_H(\pi_H - \pi_L)} \right] \\ & \geq \frac{[1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H}{4} \left[1 - \frac{[1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H}{2\pi_H} \right] \text{ for } \hat{R}_L = 1. \end{aligned} \quad (3.4)$$

Inequalities (3.3) and (3.4) define the relations between $\hat{P}_H$ and $\hat{R}_H$ given $\hat{R}_L$ so as to deter firm H from mimicking. They can be expressed as

$$\hat{P}_H \leq \bar{\bar{P}}_H(\hat{R}_H, 0; \pi_L) \equiv \frac{1}{[1 - (1 - \pi_H) \hat{R}_H]} \frac{4\pi_H^2(\pi_H^2 - \pi_L^2) - 1}{2\pi_H(2\pi_H - \pi_L)(\pi_H + \pi_L) - \pi_L^2(\pi_H - \pi_L)}; \text{ and}$$

$$\hat{P}_H \leq \bar{\bar{P}}_H(\hat{R}_H, 1; \pi_L) \equiv \frac{1}{[1 - (1 - \pi_H) \hat{R}_H]} \frac{6\pi_H(\pi_H - \pi_L)}{(7\pi_H - 3\pi_L)}, \text{ where the second argument in } \bar{\bar{P}}_H$$

is the value of $\hat{R}_L$. In particular, $\bar{\bar{P}}_H(0, 0; \pi_L) =$

$$\frac{4\pi_H^2(\pi_H^2 - \pi_L^2) - 1}{2\pi_H(2\pi_H - \pi_L)(\pi_H + \pi_L) - \pi_L^2(\pi_H - \pi_L)} \text{ and } \bar{\bar{P}}_H(1, 0; \pi_L) = \frac{1}{\pi_H} \bar{\bar{P}}_H(0, 0; \pi_L); \text{ and}$$

$$\bar{\bar{P}}_H(0, 1; \pi_L) = \frac{6\pi_H(\pi_H - \pi_L)}{7\pi_H - 3\pi_L} \text{ and } \bar{\bar{P}}_H(1, 1; \pi_L) = \frac{1}{\pi_H} \bar{\bar{P}}_H(0, 1; \pi_L).$$

To express the necessary conditions of a separating equilibrium in a more compact form, I define two subsets of the admissible space of price and refund policy.

Definition 3.3. $B^S(\pi_L) \equiv \{(P, R): P \in [0, 1], P \geq \bar{P}_H(R)\}$ and $T^S(\pi_L, R_L) \equiv \{(P, R): P \in [0, 1], P \leq \bar{P}_H(R, R_L)\}$.

Combining Lemma 3.7, Lemma 3.8, and Definition 3.3, the necessary conditions of a separating equilibrium are stated in Lemma 3.9. In particular, the strategy of firm H must be in the intersection of $B^S(\pi_L)$ and $T^S(\pi_L, R_L)$, which ensures no firm has an incentive to mimic the strategy of the other firm.

Lemma 3.9. *Necessary conditions for $(\hat{P}_H, \hat{R}_H) \neq (\hat{P}_L, \hat{R}_L)$ to be a separating equilibrium profile are:*

- (1) $(\hat{P}_H, \hat{R}_H) \in B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$,
- (2) $(\hat{P}_L, \hat{R}_L)$ is either $((\pi_L/(2\pi_H)) [1 - (1 - \pi_H)\hat{R}_H] \hat{P}_H, 0)$ or $((1/(2\pi_H)) [1 - (1 - \pi_H)\hat{R}_H] \hat{P}_H, 1)$.

Consequently, separation is only possible if $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ is not empty. In other words, separation is only possible if there exists a price P such that

$\bar{P}_H(R) \leq P \leq \bar{P}_H(R, \hat{R}_L)$. Put differently, separation is only possible if there exists a refund

policy R such that $\bar{P}_H(R) \leq \bar{P}_H(R, \hat{R}_L)$. Numerical analysis shows that separation is

impossible for large π_L , which contrasts with the case of price-advertising strategy space studied in Hertzendorf and Overgaard (1998b), where separation is possible for all π_L .

Lemma 3.10. $B^S(\pi_L) \cap T^S(\pi_L, 0)$ is empty for $\pi_L > 0.1561$ and $B^S(\pi_L) \cap T^S(\pi_L, 1)$ is empty for $\pi_L > 0.6$.

Lemma 3.10 points out that, if firm L chooses not to offer refunds, firm H has no incentive to separate from firm L, unless the quality difference is sufficiently large. On

the other hand, if firm L chooses to offer refunds, the range of the quality difference that firm H cannot separate from firm L is smaller. The cost of deterring firm L from mimicking is so high that firm H would rather to pool with firm L. The incentive of firm L to offer refunds will be discussed later.

So far, I only establish that a separating equilibrium is possible for those π_L such that $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ is nonempty. In fact, it can be shown that, for those π_L such that $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ is nonempty, any pair of strategies satisfying Lemma 3.9 can be paired with a system of consumer beliefs to constitute a separating sequential equilibrium.

Theorem 3.1. *For small π_L such that $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ is nonempty, any pair of strategies such that $(\hat{P}_H, \hat{R}_H) \in B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ and $(\hat{P}_L, \hat{R}_L) = ((\pi_L/(2\pi_H)) [1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H, 0)$ or $((1/(2\pi_H))[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H, 1)$ can be paired with a system of consumer beliefs to constitute a separating sequential equilibrium.*

Note that the system of consumer beliefs to prove the result in Theorem 3.1 contains out-of-equilibrium beliefs defined such that, given the nondeviating of the other firm, any deviating firm is viewed as offering a low-quality product. This creates the greatest possible incentive for a firm not to deviate. Whether this system of consumer beliefs makes sense or not will be discussed later in the section of equilibrium refinements.

3.3.3.3. The pooling sequential equilibria

Let us turn to pooling sequential equilibria. Before proceeding, I define another two subsets of the admissible space of price and refund policy.

Definition 3.4. $B(\pi_L) \equiv \{(P, R): P \in [0, 1], P < \bar{P}_H(R)\}$ and $B^P(\pi_L) \equiv \{(P, R): P \in [0, 1],$

$$P < P^i(R, \pi_L) \equiv \frac{1}{\frac{[1 - (1 - \pi_H)R]}{2(\pi_H - \pi_L)} + \frac{2[1 - (1 - \frac{\pi_H + \pi_L}{2})R]}{\pi_H + \pi_L}}\}.$$

Note that $B^S(\pi_L) \cup B(\pi_L) = [0, 1] \times \{0, 1\}$. That is, $B(\pi_L)$ is the closure of the complement of $B^S(\pi_L)$. Moreover, $B(\pi_L) \supseteq B^P(\pi_L)$, because $\pi_L[1 - (1 - \pi_H)R] \leq \pi_H[1 - (1 - \pi_L)R]$ and thus $P^i(R, \pi_L) \leq \bar{P}_H(R)$. With the same system of consumer beliefs, I show that every pair of strategy in the set of $B^P(\pi_L)$ can constitute a pooling sequential equilibrium.

Theorem 3.2. *For all $\pi_L \in (0, \pi_H)$, any pair of strategy such that $\Gamma_H = \Gamma_L = (\hat{P}, \hat{R}) \in B^P(\pi_L)$ can be paired with a system of consumer beliefs to constitute a pooling sequential equilibrium.*

In contrast with the case of price-advertising strategy space studied in Hertzendorf and Overgaard (1998b), where any pair of strategy in the set of $B(\pi_L)$ can constitute a pooling sequential equilibrium, the set of pooling sequential equilibrium in the case of price-refund strategy space is smaller.

Finally, let us examine if there exist separating sequential equilibria in the price-alone strategy space, which will contrast the effect of the introduction of refund policy. It is a special case of Theorem 3.1, and Lemma 3.10 has hinted the answer.

Theorem 3.3. *There exists separating sequential equilibria of the form $\Gamma_H = (\hat{P}_H, 0)$ and $\Gamma_L = ((\pi_L / (2\pi_H)) \hat{P}_H, 0)$ if and only if $\pi_L < 0.1561$.*

Theorem 3.3 demonstrates that price-alone separating sequential equilibria can only exist for sufficiently large degree of vertical differentiation. As stated in Lemma

3.10, spanning the strategy space by introducing refund policy may not span the set of separating sequential equilibria to smaller degree of vertical differentiation, depending on the refund policy of firm L. That is, if firm L chooses to offer refunds, then separating sequential equilibria exist for $\pi_L < 0.6$. However, if firm L still chooses not to offer refunds, then separating sequential equilibria exist for $\pi_L < 0.1561$, the same as if refund policy is not introduced. In fact, if firm L chooses not to offer refunds and $\pi_L < 0.1561$, separating sequential equilibria exist by firm H chooses not to offer refunds, either.

3.3.4. Equilibrium Refinements

Like the large literature on the monopoly signaling game, the analysis in section 3.3.3. shows that there exists a multiplicity of both separating and pooling sequential equilibria for this game as well. Thus, it is necessary to apply notions of equilibrium refinements to delete unreasonable sequential equilibria. In the standard one-sender-and-one-receiver signaling game, researchers usually make an appeal to such well-known refinements as the Intuitive Criterion by Cho and Kreps (1987) and the Criterion of Universal Divinity by Banks and Sobel (1987). However, it is not clear how these refinements can be directly used in the two-sender-and-one-receiver signaling game discussed in this essay. This problem has been noted in Hertzendorf and Overgaard (1998a and 1998b).

The purpose of this section is to refine multiple separating and pooling equilibria using the refinements proposed in Hertzendorf and Overgaard (1998a and 1998b).

3.3.4.1. Separating equilibria

The separating sequential equilibrium profiles characterized in section 3.3.3. are briefed as follows. Consumers expect to observe $((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L))$, where $(\hat{P}_H, \hat{R}_H) \neq$

$(\hat{P}_L, \hat{R}_L)$. In particular, $(\hat{P}_H, \hat{R}_H) \in B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ and $(\hat{P}_L, \hat{R}_L) = ((\pi_L/(2\pi_H))$
 $[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H, 0)$ or $((1/(2\pi_H))[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H, 1)$. The associated equilibrium
 beliefs are $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L))=1$, while the associated out-of-equilibrium beliefs are
 $\mu((\hat{P}_H, \hat{R}_H), (P, R))=1$ for all $(P, R) \notin \{(\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)\}$ and $\mu((P, R), (\hat{P}_L, \hat{R}_L))=0$ for
 all $(P, R) \notin \{(\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)\}$. In other words, it requires consumers to view any
 deviating firm as firm L.

Such pessimistic beliefs may not be sensible for all cases. For example, to require
 $\mu((P, R), (\hat{P}_L, \hat{R}_L))=0$ for all $(P, R) \notin \{(\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)\}$ is in fact to require consumers
 put all probability weight on the consistent reconstruction of play involving two
 simultaneous deviations from the putative profile and no probability weight on the
 consistent reconstruction of play involving one deviation from the putative profile. In
 other words, suppose *ex ante* consumers expect firm H to choose $(\hat{P}_H, \hat{R}_H)$ with
 probability 1 and firm L to choose $(\hat{P}_L, \hat{R}_L)$ with probability 1. Then, $((P, R), (\hat{P}_L, \hat{R}_L))$ is
 observed and consumers believe that firm L chooses (P, R) with probability 1, while firm
 H chooses $(\hat{P}_L, \hat{R}_L)$ with probability 1. No weight is assigned to the case where firm L
 still chooses $(\hat{P}_L, \hat{R}_L)$ and it is only firm H which deviates to (P, R) .

Without assuming the possibility of correlated deviations, this system of out-of-
 equilibrium beliefs seems implausible. To change this situation, as in Hertzendorf and
 Overgaard (1998a and 1998b), I shall propose another selection procedure that put
 probability weight on reconstruction of play consistent with a minimal number of

deviations from the putative equilibrium profile.¹⁷ This leads to the following definition.

Definition 3.5. REDE (Resistance of Equilibrium Defections) *An equilibrium profile*

$((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L))$ *is resistant to equilibrium defections if beliefs satisfy* $\mu((P^j, R^j), (P^i, R^i)) = 1$, *where* $(P^j, R^j) \neq (P^i, R^i)$, *as long as*

$$(1) (P^j, R^j) \in B^S(\pi_L) \cap T^S(\pi_L, R^j)$$

$$(2) (P^j, R^j) = ((\pi_L / (2\pi_H)) [1 - (1 - \pi_H) \tilde{R}] \tilde{P}, 0) \text{ or } ((1 / (2\pi_H)) [1 - (1 - \pi_H) \tilde{R}] \tilde{P}, 1) \text{ for some}$$

$$(\tilde{P}, \tilde{R}) \in B^S(\pi_L) \cap T^S(\pi_L, R^j).$$

The idea of REDE is as follows. *Ex ante*, consumers expect to observe $(\hat{P}_H, \hat{R}_H)$ from firm H and $(\hat{P}_L, \hat{R}_L)$ from firm L, where $(\hat{P}_H, \hat{R}_H) \neq (\hat{P}_L, \hat{R}_L)$. Instead, they observe (P^i, R^i) and (P^j, R^j) . If (P^i, R^i) is consistent with some separating equilibrium strategy played by firm H, and (P^j, R^j) is consistent with another separating equilibrium strategy played by firm L, then consumers believe that firm H chooses (P^i, R^i) with probability 1 and firm L chooses (P^j, R^j) with probability 1. In other words, beliefs are resistant to deviations to alternative sequential equilibrium pairs.

Such an interpretation is only reasonable if the sets of possible separating equilibrium strategies for firm H and firm L are disjoint. That is, the two sets are disjoint if, when the two firms choose the same refund policy, the price range for each firm to constitute separating equilibria should not overlap with each other. This will be true if, for the same refund policy, the maximum price for firm L is lower than the minimum price for firm H or vice versa. The following lemma demonstrates that this is the case.

¹⁷ See Bagwell and Ramey (1991) and Hertzendorf and Overgaard (1998a, 1998b) for a more formal discussion of consistent reconstruction of play.

Lemma 3.11. *The sets of possible separating equilibrium strategies for the two firms are disjoint.*

Since the firms are assumed to be unable to make correlated deviations at the same time, it implies the observation $((P^i, R^i), (P^j, R^j))$, where $(P^i, R^i) \notin ((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L))$ and $(P^j, R^j) \in ((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L))$, i.e., one contract is in the putative profile and the other is not. Then, suppose that $(P^j, R^j) = (\hat{P}_H, \hat{R}_H)$ and (P^i, R^i) is consistent with some separating sequential equilibrium strategy of firm L, then REDE requires consumers to believe with probability 1 that it is firm L deviating. Similarly, suppose that $(P^i, R^i) = (\hat{P}_L, \hat{R}_L)$ and (P^j, R^j) is consistent with some separating sequential equilibrium strategy of firm H, then REDE requires consumers to believe with probability 1 that it is firm H deviating. This illustrates that REDE puts all probability weight on reconstruction of play consistent with minimal number of deviations. Note that other than these two cases, REDE is silent. In this situation, then I resume the pessimistic beliefs. In particular, this implies that $\mu((P^i, R^i), (\hat{P}_L, \hat{R}_L)) = 0$ for all $(P^i, R^i) \notin B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$.

It is clear that the imposition of REDE does not change the optimal strategy of firm L. Any deviation by firm L from $(\hat{P}_L, \hat{R}_L)$ to $(P, R) \neq (\hat{P}_H, \hat{R}_H)$ still makes consumers believe that it is firm L. Although the deviation by firm L to $(P, R) = (\hat{P}_H, \hat{R}_H)$ could change consumer beliefs to $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_H, \hat{R}_H)) = 1/2$, it is suboptimal by the construction of separating equilibria. Therefore, the deviation of firm L is not a concern any more.

On the other hand, the imposition of REDE changes dramatically the optimal behavior of firm H. It is no longer that any of its deviation be viewed as offering low quality. In stead, consumer beliefs will not change as long as its deviated strategy is in the set of separating sequential equilibrium strategies for firm H, i.e., $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$. This suggests the possible deviation of firm H only occur in the set of $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$. The following lemmas show the destabilization of almost all points but two points in the set of $B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$.

Lemma 3.12. *Suppose $\pi_L < 0.1561$, then any separating profile $((\hat{P}_H, \hat{R}_H), (\hat{P}_L, 0))$ satisfying either one of the following conditions is destabilized:*

- (1) $(\hat{P}_H, \hat{R}_H) = (\hat{P}_H, 0)$ where $\hat{P}_H > \bar{P}_H(0; \pi_L)$,
- (2) $(\hat{P}_H, \hat{R}_H) = (\hat{P}_H, 1)$ where $\hat{P}_H > \bar{P}_H(1; \pi_L)$.

Lemma 3.13. *Suppose $\pi_L < 0.6$, then any separating profile $((\hat{P}_H, \hat{R}_H), (\hat{P}_L, 1))$ satisfying either one of the following conditions is destabilized:*

- (1) $(\hat{P}_H, \hat{R}_H) = (\hat{P}_H, 0)$ where $\hat{P}_H > \bar{P}_H(0; \pi_L)$,
- (2) $(\hat{P}_H, \hat{R}_H) = (\hat{P}_H, 1)$ where $\hat{P}_H > \bar{P}_H(1; \pi_L)$.

For each refund policy that firm L may choose, Lemma 3.12 and Lemma 3.13 leaves only two strategies as candidates for separating equilibrium strategies of firm H, namely, $(\hat{P}_H, \hat{R}_H) = (\bar{P}_H(0; \pi_L), 0)$ or $(\bar{P}_H(1; \pi_L), 1)$. Given $\hat{R}_L$, comparing the profits of firm H associated with these two strategies yields the optimal separating equilibrium strategy for firm H, which is reported in the following two theorems.

Theorem 3.4. *With $\pi_L < 0.1561$ for $\hat{R}_L = 0$, the unique separating equilibrium profile*

which survives REDE is $(\hat{P}_H, \hat{R}_H) = (\bar{P}_H(0, \pi_L), 0)$, $(\hat{P}_L, \hat{R}_L) = ((\pi_L/(2\pi_H)) \bar{P}_H(0, \pi_L), 0)$.

Theorem 3.5. With $\pi_L < 0.60$ for $\hat{R}_L = 1$, the unique separating equilibrium profile which survives REDE is

(1) if $\pi_L \leq 0.3$, then $(\hat{P}_H, \hat{R}_H) = (\bar{P}_H(0, \pi_L), 0)$, $(\hat{P}_L, \hat{R}_L) = ((1/(2\pi_H)) \bar{P}_H(0, \pi_L), 1)$.

(2) if $\pi_L \geq 0.3$, then $(\hat{P}_H, \hat{R}_H) = (\bar{P}_H(1, \pi_L), 1)$, $(\hat{P}_L, \hat{R}_L) = ((1/2) \bar{P}_H(1, \pi_L), 1)$.

Theorem 3.4 and Theorem 3.5 demonstrate that while it is possible to observe only firm L offering refunds, it is impossible to observe only firm H offering refunds. This is because it is too costly for firm H to deter the mimicking of firm L when firm H offers refunds. Should this be the case, firm H would rather pool with firm L than separate from firm L. More interestingly, Theorem 3.4 and Theorem 3.5 provide a rule of thumb for consumers in determining which firm offers high quality. That is, the price for a high quality good is higher. Specifically, define as $M_q \equiv [1 - (1 - \pi_q)R_q]P_q$ the expected price a consumer has to pay for the contract (P_q, R_q) of firm q, $q \in \{H, L\}$. Then, the ratio of the expected price of good H to that of good L, i.e., M_H/M_L is $2\pi_H/\pi_L$. Comparison reveals that this ratio is the same as that under complete information. Whether prices under incomplete information are distorted upward or downward from those under complete information will be discussed later.

3.3.4.2. Pooling equilibria

It is evident that REDE has no power in pruning pooling sequential equilibria. The two strategies in a pooling equilibrium profile are the same and the sets of possible pooling equilibrium strategies for the two firms are completely joint. Hence, REDE has no use and another selection procedure needs to be developed.

Hertzenndorf and Overgaard (1998b) offer a heuristic argument for reducing the set of pooling sequential equilibria. This argument, called *the incentive argument*, suggests that consumers should put all probability weight on the firm with stronger incentive of deviation. This notion of equilibrium refinement has some flavor of the Criterion of Universal Divinity. Since the structures of profit functions are the same across firms in their model, both firms have the same incentive to deviate. Then, consumers are still confused as to which firm deviating. Thus, they argue that consumers should revert to their prior beliefs, i.e., $\mu=1/2$. That is, consumers do not change their beliefs if they observe one deviating strategy. This contrasts with the pessimistic out-of-equilibrium beliefs used in looking for pooling sequential equilibria, which simply requires consumers to believe the deviating firm offering low quality. In their model, under the incentive argument, it turns out that each firm will undercut the price of its rival until matching its marginal cost. In other words, the standard zero-profit Bertrand equilibrium yields.

Now, apply the incentive argument to my model. When consumers observe one firm deviating to (P, R) from the putative pooling equilibrium $(\hat{P}, \hat{R})$, they concentrate all probability weight on the firm with stronger incentive of deviation. While if both firms have the same incentive of deviation, they revert to their prior beliefs. Since the deviating profit of a firm is $[1-(1-\pi_q)R]P$ -Demand, $q \in \{H, L\}$, the incentive of deviation will be the same for both firms if $R=0$; while it will be stronger for firm H if $R=1$, due to the higher profits for firm H. Hence, the beliefs μ for the firm deviated is $1/2$ if $R=0$ and 1 if $R=1$. The following theorem shows the nonexistence of any pooling equilibrium surviving the incentive argument due to the beliefs in the latter case.

Theorem 3.6. *There exists no pure-strategy pooling equilibrium for any $\pi_L \in (0, \pi_H)$, given the incentive argument.*

The result of Theorem 3.6 differs from that of the price-advertising model studied in Hertzendorf and Overgaard (1998b) in that the quality level enters into the profit function of the firm offering refunds, but not of the firm offering no refund. Whereas it does not show up in the profit functions of firms in the price-advertising model no matter what the advertising level is.¹⁸

One should be cautious that no pooling equilibrium surviving the incentive argument never means that there exists no pooling equilibrium surviving other arguments. For instance, Hertzendorf and Overgaard (1998a) propose the Pareto-Perfection argument, which suggests that firms pick the strategy preferred by both firms.¹⁹ It is as if firms can coordinate implicitly *ex ante*. Obviously, a pooling equilibrium exists given the Pareto-Perfection argument if there is a strong commonality of preferences across firms, which is the case when two firms have the same structure of profit functions. Applying this argument to my model, I can find the optimal pooling price for any given refund policy. However, each firm has different preferences about refund policy. In particular, firm H prefers the strategy set with offering refunds, while firm L prefers the one without offering refunds. Hence, under the Pareto-Perfection

¹⁸ More specifically, the advertising in the price-advertising model studied in Hertzendorf and Overgaard (1998b) is a dissipative signal, in the sense that it does not affect the demand function, only transmit a signal.

¹⁹ See, for example, Bagwell and Ramey (1988), and more indirectly, Mailath et al. (1993) and Overgaard (1993).

argument, there are two pooling equilibria, which does not help suggesting the playing rule for each firm, either.

In summary, if the REDE and the incentive argument are accepted, there is a unique separating equilibrium for all π_L in $(0, 0.1561)$ if firm L offers refunds and for all π_L in $(0, 0.6)$ if firm L offers no refund. Other than these two ranges of π_L , there is no suggestion about the appropriate behaviors of firms. From this characterization, one may be curious about the optimal refund policy of firm L for quality π_L in the range of $(0, 0.1561)$, because firm L has two choices. Comparison of the profits of firm L for these choices reveals that firm L is indifferent between offering refunds and not, since profits are the same.

3.3.5. Comparisons

This section examines the distortion of the prices compared to those under complete information. Before proceeding, recall the notations P_{q0}^* and P_{q1}^* , where $q \in \{H, L\}$, given in the section of complete information. Similarly, denote as $\hat{P}_{q0}$ the incomplete information price of firm q offering no refund, and as $\hat{P}_{q1}$ the incomplete information price of firm q offering refunds, $q \in \{H, L\}$. Let us first determine the distortion in the no-refund strategy space, i.e., when neither firm can offer a refund.

Corollary 3.1. *In any no-refund separating equilibrium, $\hat{P}_{H0} > P_{H0}^*$ and $\hat{P}_{L0} > P_{L0}^*$.*

Corollary 3.1 presents the upward distortion of both prices compared to those under complete information. This result is merely the same as in Hertzendorf and Overgaard (1998a and 1998b), because without offering refunds or advertising the three models collapse into one. It distinguishes from those of the literature on monopoly models of price signaling, where most results show that the price of the high quality good

is distorted upwards with the price of the low quality good undistorted for any degree of vertical differentiation.

Next, let us examine the distortion of prices in the price-refund strategy space, i.e., when either firm can offer a refund, focusing on the refined separating equilibria. Notice that there are two equilibrium prices of either quality in the complete information, namely, P_{q0}^* and P_{q1}^* , $q \in \{H, L\}$, and the comparison should be made under the same refund policy. That is, $\hat{P}_{qi}$ should be compared to P_{qi}^* , not P_{qj}^* , where $q \in \{H, L\}$, $i \neq j$, i and $j \in \{0, 1\}$.

Corollary 3.2. *At each refined separating equilibrium profile, both prices under incomplete information are distorted upward for any refund policy chosen by firm L.*

Corollary 3.2 demonstrates an upward distortion of prices of both qualities for any degree of quality differentiation if the separating equilibrium exists. This upward distortion is the effect of information asymmetry between consumers and producers. For the same set of refund policies as that under complete information, the set of prices in a separating equilibrium has to be distorted so as to deter each firm from mimicking the pricing of the other firm. A downward distortion for firm H is not likely to reach this goal, because more consumers could accept the pooling situation and purchase either good, which gives firm L more incentive to pool with firm H and to share half of market demand.²⁰ Thus, firm H has to distort its price upward. It follows that firm L has to distort its price upward as well, according to its best reply function. It is noteworthy that the upward distortion for firm L in duopoly, which contrasts with the nondistortion for

²⁰ Note that under complete information, the demand of firm L is only half of the demand of firm H.

firm L in monopoly, is an effect of competition between the two firms.

Moreover, define $\hat{M}_q \equiv [1 - (1 - \pi_q)\hat{R}_q]\hat{P}_q$ and $M_q^* \equiv [1 - (1 - \pi_q)R_q^*]P_q^*$, $q \in \{H, L\}$,

which are the expected prices a consumer pays to respective producers under different information structures. The following corollary examines the distortion of $\hat{M}_q$ from M_q^* , $q \in \{H, L\}$, which is a more meaningful question.

Corollary 3.3. *At each refined separating equilibrium profile, both of the expected prices under incomplete information are distorted upward for any refund policy chosen by firm L.*

Corollary 3.3 points out the signaling cost incurred by each firm in duopoly, which contrasts with the costless signal in monopoly studied by Shieh (1996) and Levy (1997) and the same as the homogeneous-consumer case. Moreover, Corollary 3.3 implies that the expected utility of each consumer is (weakly) lower under incomplete information than under complete information.²¹ In sum, none of the two firms and consumers strictly benefits from this information asymmetry, contrasting with the homogeneous-consumer case, where consumers get higher expected utilities under incomplete information in the separating equilibrium if one exists.

²¹ To see this, note that $\hat{D}_H < D_H^*$, $\hat{D}_L > D_L^*$, and $1 - \hat{D}_H - \hat{D}_L > 1 - D_H^* - D_L^*$. It is easy to check that a consumer who buys the product of the same quality under both information structures is worse off under incomplete information. A consumer who does not buy under either information structure has the same expected utility. Obviously, a consumer who buys the low-quality good under complete information and does not buy under incomplete information is worse off under incomplete information. A consumer who buys the high-quality good under complete information and buys the low-quality good under incomplete information is worse off under incomplete information, since $\pi_H r - M_H^* > \pi_L r - M_L^* > \pi_L r - \hat{M}_L$.

3.4. Conclusion And Extensions

Under complete information, when two firms compete in Bertrand competition with risk-neutral consumers, whether or not refunds are offered is irrelevant. Obviously, there is no need for firms to signal quality through refunds when the information is already known by consumers. This result is robust to the valuations of consumers and to the timing of firm decisions of prices and refund policies.

When quality is unobservable, the two cases yield different results. In the homogeneous-consumer case, in equilibrium, the high-quality firm cannot signal its quality simply through price. Instead, it is forced in equilibrium to pool with the low-quality firm at a high price at most equal to the consumer's average valuation. Thus, the high-quality firm may have an incentive to signal its quality if its profits are lower than under complete information. In addition to such strategies as advertising, warranties, and best-price guarantees, a Money-back guarantee is a possible instrument.²² If only one firm offers refunds, the refund itself is a signal of high quality. But, it is a costly signal in that the price is distorted downwards and the profits are still lower than under complete information for the high-quality firm. Moreover, the high-quality firm may not find it in its best interest to signal its quality. Both cases where neither firm offers a refund and where both firms offer refunds are possible pooling equilibria under respective conditions of parameters.

Furthermore, if only the high-quality firm offers a refund, the level of social welfare is the same in equilibrium with incomplete information as with complete

²² Milgrom and Roberts (1986) offer a signaling model in which the informed monopolist uses advertising and price to signal its quality to uninformed heterogeneous consumers. See Levy (1997) about best-price guarantees.

information. The differences are that, in equilibrium, consumers get higher expected utility while the profits of the high-quality firm is lower under incomplete information than under complete information. The low-quality firm is not affected and has to exit the market because of zero demand.

In the heterogeneous-consumer case, none of the complete information equilibrium profiles can constitute a separating equilibrium when incomplete information is present. Further, I illustrate the existence of separating sequential equilibria for a sufficiently large degree of vertical differentiation and the existence of pooling sequential equilibria for any degree of vertical differentiation. I find that enabling the choice of the refund policy expands the range of the parameter π_L for the existence of separating equilibria.

Applying the REDE to reduce the set of multiple separating equilibria, I may or may not obtain a unique refined separating equilibrium depending on the parameter π_L . More surprisingly, the case of $(\hat{R}_H, \hat{R}_L) = (0, 1)$ can be a separating equilibrium for a sufficiently large degree of vertical differentiation. I also show that no pooling equilibrium survives the refinement of the incentive argument for any parameter π_L .

Compared to the equilibrium price under complete information, the equilibrium prices under incomplete information are distorted upwards, so are the expected prices. This implies that signaling via money-back guarantees in duopoly is costly. Moreover, consumers are weakly worse off under incomplete information, contrasting with the homogeneous-consumer case.

In reality, we do observe cases in which only one firm offers a refund, cases in which neither firm offers a refund, and cases in which both firms offer refunds, consistent

with the results here.²³ However, we do not know if the case of only one firm offering refunds fits more to the prediction of the first case or the second case. Thus, it raises a need of an empirical study about whether the only firm offering refunds is the high-quality firm or the low-quality firm, supposing that the two firms offer the same product of different quality.

Another factor not considered in this model is the moral hazard problem which arises, for example, when consumers can put effort to affect the probability that a product works.²⁴ In this situation, the probability that a product works may be higher or lower than what it should be. Thus, full refunds may not be appropriate. To do this would require assuming risk-averse consumers.²⁵

There is a gap needed to be filled in between the homogeneous-consumer case and the heterogeneous-consumer case. That is, the effects of different cost structures and different timing of firm decisions of prices and refund policies should be investigated. Only after that can we find the reasons making the results of the two cases different.

The model in the first case can be extended by considering the cost-reducing investment. Intuitively, the investment incentive can be weakened when product quality is unobservable before purchase because consumers do not know whether a lower price

²³ For example, in the Wal-mart, Brita's water filter is sold without money-back guarantee, while the other brand offers money-back guarantee; in Columbia, Taiwan, or Turkey, there is no practice of money-back guarantee; and in the U.S., many stores offers money-back guarantees for all of the various products they sell.

²⁴ See Emons (1988), Mann and Wissink (1988), Lutz (1989), and Mann and Wissink (1990).

²⁵ See Che (1996) for a study of customer return policies for experience goods in a monopoly with risk averse consumers.

comes from lower costs or lower quality. Shieh (1993) examines the incentives of a monopolist for cost-reducing investment in a signaling model of product quality. She finds that asymmetric information about quality may strengthen or weaken a firm's incentive to adopt a process innovation, depending on whether low-price or high-price signaling is used. It would be interesting to examine if the results still hold in duopoly.

The model in the second case can be extended in several ways. The first is to consider the transaction costs of redeeming a money-back guarantee incurred by buyers or sellers. Moorthy and Srinivasan (1995) discuss this type of model in monopoly. They show that money-back guarantees signal quality by exploiting the higher probability of returns for a low-quality good, and the attendant higher transaction costs. However, if the seller's transaction costs are very large, then there are less costly ways to signal, namely, charging a high price. They compare the signaling performance of price itself and price-refunds and find that under certain conditions a refund is necessary to signal and that under other conditions a refund is a useful supplement to price. These conclusions are drawn for a monopoly. It would be interesting to see if they are robust for a duopoly.

The second extension is to consider entry deterrence. Suppose the two firms enter the market at different time. Although consumers may have experienced and known the product quality of the incumbent, they have no idea about the product quality of the entrant and need to make an inference from prices or other characteristics of products. Bagwell and Ramey (1991) discuss such issue with all firms producing products of the same quality. It would make a contribution to the literature of limit pricing if considering this issue with firms producing products of different quality.

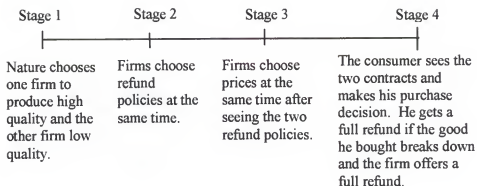


Figure 3.1. The Time Line Of The Model In The Homogeneous-Consumer Case

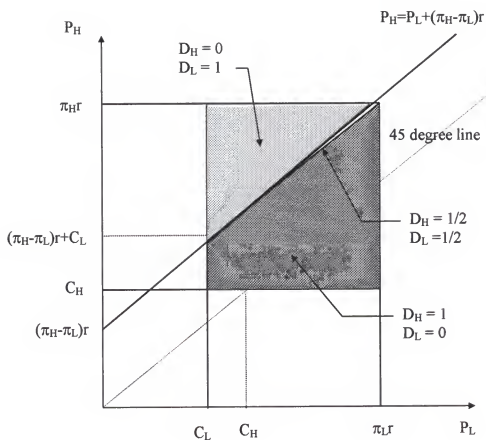


Figure 3.2. Bertrand Competition In Duopoly When Quality Is Observable and Neither Firm Offers A Refund

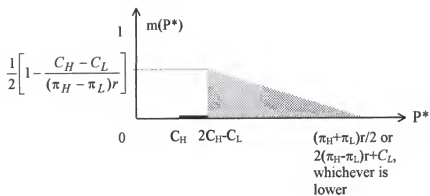


Figure 3.3. The Relation Between Out-Of-Equilibrium Belief $m(P^*)$ And P^* When Neither Firm Offers A Refund And $2C_H - C_L < (\pi_H + \pi_L)r/2$

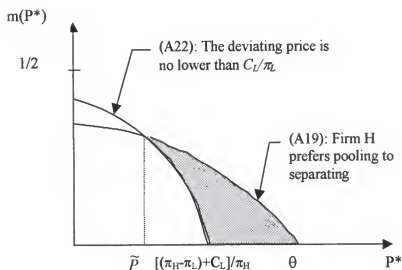


Figure 3.4. The Relation Between Out-Of-Equilibrium Beliefs $m(P^*)$ And Pooling Equilibrium Price P^* When Both Firms Offer Refunds

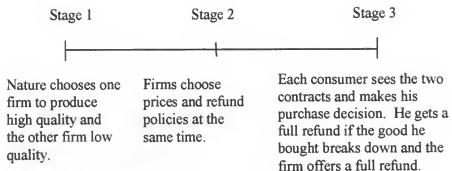


Figure 3.5. The Time Line Of The Model In The Heterogeneous-Consumer Case

Table 3.1. Summary Of The Equilibrium Under Alternative Refund Strategies

	(0,0)		(1,0)	(0,1)	(1,1)	
	$\alpha > 0$	$\alpha < 0$			$\beta > 0$	$\beta < 0$
P_H	$(\pi_H + \pi_L)r/2$	$2(\pi_H - \pi_L)r + C_L$	$C_L/\pi_L - \varepsilon$	$(\pi_H - \pi_L)r + C_L - \varepsilon$	$C_L/\pi_L - \varepsilon$	θ
P_L	$(\pi_H + \pi_L)r/2$	$2(\pi_H - \pi_L)r + C_L$	C_L	C_L/π_L	$[C_L/\pi_L, (2C_L/\pi_L - C_H/\pi_H)]$	θ
Π_H	$(\pi_H + \pi_L)r/4 - C_H/2$	$(\pi_H - \pi_L)r + [C_L - C_H]/2$	$\pi_H[C_L/\pi_L - C_H/\pi_H]$	$(\pi_H - \pi_L)r - (C_H - C_L)$	$\pi_H[C_L/\pi_L - C_H/\pi_H]$	$(\pi_H\theta - C_H)/2$
Π_L	$(\pi_H + \pi_L)r/4 - C_L/2$	$(\pi_H - \pi_L)r$	0	0	0	$(\pi_L\theta - C_L)/2$
EU	0	$(-3\pi_H + 5\pi_L)r/2 - C_L$	$(\pi_Lr - C_L)\pi_H/\pi_L$	$\pi_Lr - C_L$	$(\pi_Lr - C_L)\pi_H/\pi_L$	$(r - \theta)(\pi_H + \pi_L)/2$

Note: The first and second element in (...) is the refund strategy of firm H and firm L, respectively. "0" means the firm offers no refund, and "1" means the firm offers a full refund. $\alpha \equiv \{2(\pi_H - \pi_L)r + C_L - (\pi_H + \pi_L)r/2\}$, $\beta \equiv \{(2\pi_H - \pi_L)C_L/\pi_L - C_H - (\pi_H -$

$$\pi_L)r\}, \text{ and } \theta \equiv \frac{2(\pi_H - \pi_L)r + \pi_L \frac{C_H}{\pi_H}}{2\pi_H - \pi_L}.$$

Table 3.2. Summary Of Equilibrium Refund Strategies For Alternative Combinations Of Parameters

		$\beta > 0$	$\beta < 0$
$\alpha > 0$	$\eta > 0$	$(0, 0)_P$	$(0, 0)_P$ and $(1, 1)_P$
	$\eta < 0$	$(1, 0)_S$	$(1, 1)_P$
$\alpha < 0$		$(0, 0)_P$	$(0, 0)_P$

Note: The first and second element in $(\cdot, \cdot)_i$ is the equilibrium refund strategy of firm H and firm L, respectively. "0" means the firm offers no refund, and "1" means the firm offers a full refund. The subscript $i=S$ means a separating equilibrium; $i=P$ means a pooling equilibrium. $\alpha \equiv \{2(\pi_H - \pi_L)r + C_L - (\pi_H + \pi_L)r/2\}$, $\beta \equiv \{(2\pi_H - \pi_L)C_L/\pi_L - C_H - (\pi_H - \pi_L)r\}$, and $\eta \equiv (\pi_H + \pi_L)/4 - C_H/2 - \pi_H(C_L/\pi_L - C_H/\pi_H)$.

Table 3.3. Summary Of Equilibrium Results

		Π_H	Π_L	EU	SW
Complete Information	Price-alone	$[(\pi_H - \pi_L)r - (C_H - C_L)]$	0	$\pi_L r - C_L$	$\pi_H r - C_H$
	Price-Refund	$[(\pi_H - \pi_L)r - (C_H - C_L)]$	0	$\pi_L r - C_L$	$\pi_H r - C_H$
Incomplete Information	Price-alone	$\alpha > 0$	$(\pi_H + \pi_L)r/4 - C_H/2$	0	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$
		$\alpha < 0$	$(\pi_H - \pi_L)r + [C_L - C_H]/2$	$(-3\pi_H + 5\pi_L)r/2 - C_L$	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$
	Price-Refund	$\beta > 0, \eta > 0$	$(\pi_H + \pi_L)r/4 - C_H/2$	0	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$
		$\beta > 0, \eta < 0$	0	$(\pi_L r - C_L)\pi_H / \pi_L$	$\pi_H r - C_H$
		$\beta < 0, \eta > 0$	$(\pi_H + \pi_L)r/4 - C_H/2$ or $(\pi_L \theta - C_L)/2$	0 or $(r - \theta)(\pi_H + \pi_L)/2$	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$ $[(\pi_H + \pi_L)r - (C_H + C_L)]/2$
		$\beta < 0, \eta < 0$	$(\pi_H \theta - C_H)/2$	$(r - \theta)(\pi_H + \pi_L)/2$	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$
		$\beta > 0$	$(\pi_H - \pi_L)r + [C_L - C_H]/2$	$(-3\pi_H + 5\pi_L)r/2 - C_L$	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$
		$\beta < 0$	$(\pi_H - \pi_L)r + [C_L - C_H]/2$	$(-3\pi_H + 5\pi_L)r/2 - C_L$	$[(\pi_H + \pi_L)r - (C_H + C_L)]/2$

Note: $\alpha \equiv [2(\pi_H + \pi_L)r + C_L - (\pi_H + \pi_L)]C_L/\pi_L - C_H - (\pi_H - \pi_L)r$, $\beta \equiv \{(2\pi_H - \pi_L)r/2\}$, $\eta \equiv (\pi_H + \pi_L)r$, $\theta \equiv (2\pi_H - \pi_L)r/2 - \pi_H(C_L/\pi_L - C_H/\pi_H)$ and $\theta \equiv \frac{2(\pi_H - \pi_L)r + \pi_L \frac{C_H}{\pi_H}}{2\pi_H - \pi_L}$.

CHAPTER 4 CONCLUSIONS

This dissertation has examined price competition in oligopoly when suppliers are better informed than demanders. In particular, Chapter 2 sheds light on a very likely negative relationship between competition and investments of HMOs in preventive care, which is often claimed to be an important characteristic of HMOs. Moreover, Chapter 2 discusses numbers of HMOs in the free entry and in the social optimum as a policy implication.

The principal findings of Chapter 2 are as follows. First, in the unique separating equilibrium, HMOs offer each customer a premium linearly increasing with his illness level, and each customer, without knowing his own illness level, accepts the offer with a probability non-increasing with the premium at a decreasing rate. Secondly, HMOs generally underinvest, which can be mitigated by a higher verification cost, a higher utility of a customer being healthy, or a lower transferability of preventive care. However, too much general investment, used to reduce the probability of customers being severely ill, can occur when HMOs commit to the investment levels announced in their plans. Finally, HMOs can be profitable in the social optimum, which leads to too many HMOs in the free entry market.

Chapter 3 has examined the signaling role played by prices and money-back guarantees in duopoly with two different product qualities. Two cases are considered. In the first case, where consumers have the same valuation on a working product and each firm with different unit costs of production simultaneously decides a refund policy

followed by a price, three equilibria can arise. The unique separating equilibrium is that only the high-quality firm offers refunds, showing that the refund itself signals product qualities. Signaling is costly to the high-quality firm and consumers are better off.

In the second case, where consumers have different valuations and each firm with the same unit cost of production simultaneously decides a refund policy and a price, three separating equilibria can arise. In particular, the separating equilibrium with different refund policies is that only the low-quality firm offers refunds, a contrast to the first case. Signaling is also costly but consumers are weakly worse off.

APPENDIX A
PROOFS OF THE MAIN RESULTS IN CHAPTER 2

A.1. Proof of Proposition 2.1

I. Equilibrium. To see the triple (O^*, a^*, b^*) as an equilibrium, I show that: (A) given $b^*(O)$, $a^*(O)$ maximizes $\Pi^c(O, a, b)$; (B) given $a^*(O)$, $O^*(c)$ maximizes $\Pi^0(c, O, a)$; and (C) $b^*(O) \in [\underline{c}, \bar{c}] \forall O$, and $b^*(O^*(c)) = c \forall c \in [\underline{c}, \bar{c}]$.

(A) First, $\forall O \in [\underline{Q}, \bar{O}]$, $b^*(O) = H - V + \mu\phi(e) - [(H - O)/\lambda(n)]$. Then,

$$\Pi^c(O, a, b) = a[H - O] + (1 - a)\lambda(n)[H - V - (b^*(O) - \mu\phi(e))] = H - O,$$

which is independent of control variable a . Thus, $a^*(O)$ as described in Proposition 2.1 is optimal $\forall O \in [\underline{Q}, \bar{O}]$. Second, $\forall O > \bar{O}$, $b^*(O) = \bar{c}$, and

$$\Pi^c(O, a, b) = a[H - O] + (1 - a)\lambda(n)[H - V - (\bar{c} - \mu\phi(e))].$$

Then, $\partial \Pi^c / \partial a = -O + \bar{O} < 0$. Thus, $a^*(O) = 0$ is optimal $\forall O > \bar{O}$. Last, $\forall O < \bar{O}$, $b^*(O) = \underline{c}$, and $\Pi^c(O, a, b) = a[H - O] + (1 - a)\lambda(n)[H - V - (\underline{c} - \mu\phi(e))]$. Then, $\partial \Pi^c / \partial a = -O + \underline{Q} > 0$. Thus, $a^*(O) = 1$ is optimal $\forall O < \bar{O}$.

(B) First, offers of $O > \bar{O}$ are dominated by $\bar{O}$, for any $c \in [\underline{c}, \bar{c}]$. To see this,

$$\begin{aligned} \Pi^0(c, \bar{O}; a^*) - \Pi^0(c, O; a^*) &= a^*(\bar{O})[\bar{O} - c + \phi(e)] - 0[O - c + \phi(e)] \\ &= \exp\left\{\frac{\lambda(n)(\underline{c} - c)}{G}\right\} \{H - \lambda(n)[H - V - (\bar{c} - \mu\phi(e))]\} > 0, \forall O > \bar{O}. \end{aligned}$$

Thus, HMO_0 will choose $\bar{O}$ over any greater O . Similarly, offers of $O < \underline{Q}$ are dominated by $\underline{Q}$, for any $c \in [\underline{c}, \bar{c}]$. To see this,

$$\Pi^0(c, \underline{Q}; a^*) - \Pi^0(c, O; a^*) = a^*(\underline{Q})[\underline{Q} - c + \phi(e)] - 1[O - c + \phi(e)] = [\underline{Q} - O] > 0, \forall O < \underline{Q}.$$

Thus, HMO_0 will choose $\underline{Q}$ over any smaller O .

It follows, from the above arguments, that $O^*(c) \in [\underline{Q}, \bar{O}]$, for any $c \in [\underline{c}, \bar{c}]$. Then, $a^*(O)$ is twice differentiable for $O \in [\underline{Q}, \bar{O}]$. Take the first-order differentiation of $\Pi^0(c, O; a^*)$ with respect to O , and I get

$$\frac{\partial \Pi^0}{\partial O} = -\exp\left\{\frac{H - \lambda[H - V - (c - \mu\phi(e))] - O}{G}\right\} [O - (c - \phi(e)) - G],$$

which is 0 at the optimal O^* . Thus, $O^*(c) = H - \lambda(n)[H - V - (c - \mu\phi(e))]$. It can be checked that $\partial^2 \Pi^0 / \partial O^2 < 0$ at $O = O^*$, so $O = O^*(c)$ is a local maximum. It is a global maximum as well, because there is only one stationary point in the range of $[\underline{Q}, \bar{O}]$.

(C) In equilibrium, it is already showed that $O^*(c) \in [\underline{Q}, \bar{O}]$, for any $c \in [\underline{c}, \bar{c}]$. Therefore, $b^*(O^*(c)) = H - V + \mu\phi(e) - \{[H - (H - \lambda(H - V - (c - \mu\phi(e))))]/\lambda(n)\} = c$. It can be seen that, on the equilibrium path, the out-of-equilibrium beliefs specified in Proposition 2.1 are not crucial to the argument.

II. Uniqueness. The proof has seven steps, which shows that $a^*(.)$ is optimal in the class of all functions, not just differentiable ones. It also shows that there are no other equilibria involving, for example, nondifferentiable functions $a(O)$.

(A) The separating equilibrium offer-price function $O^*(c)$ must be monotone increasing.

(A1) If it were not monotone, it would not separate types.

(A2) If it were monotone decreasing, then $b^*(O)$ would also be monotone decreasing;

examination of equation (2.3) implies that in this case $a^*(.)$ must be nonincreasing. But if

$a^*(.)$ is nonincreasing, then $O^*(.)$ decreasing cannot be optimal for the HMO, because as c rises, the payoff $[O-c+\phi(e)]$ falls and is at most as likely to be accepted. In other words, the expected profits of the HMO decrease with c .

From (A1) and (A2), thus $O^*(.)$ must be monotone increasing. Denote as $\underline{Q}^*$ and $\bar{O}^*$ the lowest and highest offer price made under $O^*(.)$.

(B) Recall the definition of $\underline{Q}$ and $\bar{O}$. Since $b^*(O) \in [\underline{c}, \bar{c}]$, equation (2.3) implies that $a^*(O)=1$ for $O < \underline{Q}$ and $a^*(O)=0$ for $O > \bar{O}$.

(C) The separating equilibrium acceptance probability function $a^*(.)$ must be monotone decreasing on $[\underline{Q}^*, \bar{O}^*]$ except where $a^*(O)=0$.

(C1) $a^*(.)$ cannot be increasing on $[\underline{Q}^*, O_1]$ for some $O_1 > \underline{Q}^*$. For if it were, then the HMO of type $\underline{c}$ would strictly prefer O_1 to $\underline{Q}^*$.

(C2) The same argument implies that $a^*(.)$ cannot be constant on some neighborhood of $\underline{Q}^*$, unless $a^*(\underline{Q}^*)=0$.

From (C1) and (C2), thus, there are two cases to be considered: (i) $a^*(\underline{Q}^*)=0$. In this case $a^*(O)=0$ for all $O \in [\underline{Q}^*, \infty)$, for if $a^*(O) > 0$ for some $O > \underline{Q}^*$, then the HMO of type $\underline{c}$ would strictly prefer O to $\underline{Q}^*$. For $a^*(\underline{Q}^*)=0$, equation (2.3) implies that $\underline{Q}^* \geq \underline{Q}$. But since $a^*(O)=1$ for $O < \underline{Q}$, the HMO of type $\underline{c}$ would strictly prefer $\underline{Q}-\delta$ to $\underline{Q}^*$ for any $\delta < \underline{Q}-\underline{c}+\phi(e)$. Hence, case (i) is ruled out. (ii) $a^*(O)$ is monotone decreasing on $[\underline{Q}^*, O_1]$ for some $O_1 > \underline{Q}^*$. This implies that $a^*(O) \in (0, 1)$ for $O \in (\underline{Q}^*, O_1)$, which in turn implies that $O^*(c) = H-\lambda(n)[H-V-c+\phi(e)]$. If $a^*(\underline{Q}^*) \in (0, 1)$, then $O^*(\underline{c}) = \underline{Q}^* = \underline{Q}$. If $a^*(\underline{Q}^*)=1$, then equation (2.3) implies $\underline{Q}^* \leq \underline{Q}$. But if $\underline{Q}^* < \underline{Q}$, then the HMO of type $\underline{c}$ would prefer $\underline{Q}$ to

$\underline{Q}-\delta$ (which is also accepted with probability 1) for sufficiently small δ . Hence, $\underline{Q}^*=\underline{Q}$. If $O_1 \leq \bar{O}^*$, then for each $O \in [\underline{Q}, O_1)$ there exist $c \in [\underline{c}, \bar{c}]$ such that $O=O^*(c)$.

Repeating these arguments (e.g., $a^*(O)$ cannot increase on $[s_1, s_2]$ for some $s_2 > s_1$, and $a^*(O)$ cannot remain constant unless $a^*(O)=0$, etc.) implies that $a^*(O) \in (0, 1)$ and is monotone decreasing for $O \in (\underline{Q}, \hat{O})$, and $a^*(O)=0$ for $O \in (\hat{O}, \infty)$ for some $\hat{O}$ which might possibly be no greater than $\bar{O}^*$. Moreover, if $\hat{O} \leq \bar{O}^*$, then for each $O \in [\underline{Q}, \hat{O})$ there exists a $c \in [\underline{c}, \bar{c}]$ such that $O=O^*(c)=H-\lambda(n)[H-V-c+\phi(e)]$.

(D) The function $a^*(O)$ must be continuous for $O \in [\underline{Q}, \hat{O})$, for any jump must be downward and a downward jump at (say) O_1 would cause the HMO of type c_1 such that $O^*(c_1)=O_1$ to prefer strictly an offer of $O_1-\delta$ for sufficiently small δ . In particular, this argument implies that $a^*(\underline{Q})=1$.

(E) The function $a^*(O)$ is differentiable almost everywhere on $[\underline{Q}, \hat{O}]$. This follows from the fact that $a^*(.)$ is a decreasing function, according to Royden (1968, p. 96, Theorem 2).

(F) Suppose $a^*(O)$ is differentiable at $O \in [\underline{Q}, \hat{O})$. Since there exists an HMO of type c such that O maximizes $\Pi^0(c, O; a)$, and since $a^*(O) \in (0, 1)$, equations (2.3) and (2.5) must both hold at O . Hence, equation (2.6) must hold almost everywhere on $[\underline{Q}, \hat{O})$. Then for O in this interval, $a^*(O)=\lambda \exp\{-O/G\}$, where the continuity of $a^*(.)$ across any possible points of nondifferentiability ensures that the same constant λ applies. In other words, $a^*(O)$ is differentiable on $(\underline{Q}, \hat{O})$.

(G) I have already argued that the boundary condition $a^*(\underline{Q})=1$ is necessary, so that $a^*(O)=\exp\{(\underline{Q}-O)/G\}$ for $O \in (\underline{Q}, \hat{O})$. Note that there is no $\hat{O} < \infty$ at which this function

reaches 0. Moreover, $a^*(O)$ cannot jump to 0 for any $\hat{O} \leq \bar{O}^*$. To see this, denote as $\hat{c}$ the HMO type such that $\hat{O} = H - \lambda(n)[H - V - \hat{c} + \mu\phi(e)]$. The HMO of type $\hat{c}$ would get 0 because $a^*(\hat{O}) = 0$. However, by offering $\hat{O} - \delta$, type $\hat{c}$ would receive a lottery between $[\hat{O} - \delta - \hat{c} + \phi(e)]$ and 0, which is strictly greater for sufficiently small δ . Thus, $a^*(O) > 0$ for all $O \leq \bar{O}^*$. Hence, $\bar{O}^* = \bar{O}$. The downward jump in $a^*(\cdot)$, which occurs just after $\bar{O}$, is permissible because (in equilibrium) no HMO of any type offers $O > \bar{O}$; thus, there are no HMO of any type who might be tempted by this jump to offer less.

III. Elimination of Pure Pooling Equilibria. I use Banks and Sobel's (1987)

"universally divine equilibrium" concept to eliminate equilibria involving pooling.

(A) Consider the following class of pure pooling equilibria: "All levels of HMO₀ offer the same price O^* , which is accepted by the customer with probability 1."¹

Such equilibria exist if $H - O^* \geq \lambda(n)[H - V - E(c) + \mu\phi(e)]$ and $O^* - \bar{c} + \phi(e) \geq 0$, where $E(c)$ is the unconditional expected value of illness levels. That is, the customer must prefer accepting the offer to rejecting it, and all levels of HMO₀ (in particular, the one with the highest illness level) prefer being accepted to being rejected. Thus, such an O^* exists if and only if $\bar{c} - \lambda E(c) \leq (1 - \lambda)H + \lambda V + (1 - \lambda\mu)\phi(e)$ ($\because \bar{c} - \phi(e) \leq O^* \leq H - \lambda(n)[H - V - E(c) + \mu\phi(e)]$).

Now, the problem is to find out-of-equilibrium beliefs, $E(c|O)$ for $O \neq O^*$, that support accepting at O^* as an equilibrium. First, for $O^* = H - \lambda(n)[H - V - E(c) + \mu\phi(e)]$, the beliefs $E(c|O) = E(c)$ are sufficient. With these beliefs, any $O < O^*$ is accepted with

¹ Here and throughout the essay, "HMO₀ of level c " is the abbreviation of "HMO₀ whose customer's illness level is c ".

probability 1, while any $O > O^*$ is rejected with probability 1. Second, for $O^* < H - \lambda(n)[H - V - E(c) + \mu\phi(e)]$, we can assign beliefs such that any offer $O < O^*$ is accepted with probability 1, while any offer $O > O^*$ is rejected (with sufficiently high probability to make it undesirable) as long as $E(c|O) < H - V + \mu\phi(e) - [(H - O)/\lambda(n)]$.

But all pure pooling equilibria fail to be "universally divine", as defined by Banks and Sobel (1987). Suppose that there is a pure pooling equilibrium at O^* , and consider deviations to some price O , where $O > O^*$. Define $\theta(c; O)$ as the probability of rejecting the offer that makes an HMO₀ of level c indifferent between the equilibrium offer O^* and O . By definition, this means $[O^* - c + \phi(e)] = (1 - \theta(c; O))[O - c + \phi(e)]$, which can be rewritten as $\theta(c; O) = \frac{[O - O^*]}{O - c + \phi(e)} \in (0, 1)$. Since $\theta'(c; O) = [O - O^*]/[O - c + \phi(e)]^2 > 0$ for $O > O^*$, $\theta(c; O)$

is maximized at $c = \bar{c}$. This implies that $\bar{c}$ is the level most likely to deviate to O .

According to Banks and Sobel (1987), universal divinity requires the customer to believe that an out-of-equilibrium offer only comes from the level most likely to deviate; in this case, it requires the customer to believe that any HMO₀ which offers $O > O^*$ is of level $\bar{c}$ with probability 1. Thus, for $O < H - \lambda(n)[H - V - \bar{c} + \mu\phi(e)]$, the best response of the customer is to accept the offer with probability 1. Then, offer O will be made (i.e., deviation will definitely happen), and therefore, O^* is not universally divine.

(B) Similar arguments show that partial pooling equilibria are not universally divine, either. To see this, consider the following class of partial pooling equilibria: "A set S of levels of HMO₀ offer the same price O^* , which is accepted by the customer with probability 1, while other levels of HMO₀ offer their prices according to the separating equilibrium."

Such partial equilibria exist if, especially for the pooling part, $H-O^* \geq \lambda(n)[H-V-E(c|S)+\mu\phi(e)]$ and $O^*-c_s+\phi(e) \geq 0$, where $E(c|S)$ is the conditional expected value of illness levels over set S and c_s is the highest illness level in set S . That is, the customer must prefer accepting the offer to rejecting it, and all levels (in S) of HMO₀ (in particular, the one with the highest illness level in S) prefer being accepted to being rejected. Thus, such an O^* exists if and only if $c_s-\lambda E(c|S) \leq (1-\lambda)H+\lambda V+(1-\lambda\mu)\phi(e)$ ($\because c_s-\phi(e) \leq O^* \leq H-\lambda(n)[H-V-E(c|S)+\mu\phi(e)]$).

Now, the problem is to find out-of-equilibrium beliefs, $E(c|O)$ for $O \neq O^*$, that support accepting at O^* as an equilibrium. First, for $O^*=H-\lambda(n)[H-V-E(c|S)+\mu\phi(e)]$, the beliefs $E(c|O)=E(c|S)$ are sufficient. With these beliefs, any $O < O^*$ is accepted with probability 1, while any $O > O^*$ is rejected with probability 1. Second, for $O^* < H-\lambda(n)[H-V-E(c|S)+\mu\phi(e)]$, we can assign beliefs such that any offer $O < O^*$ is accepted with probability 1, while any offer $O > O^*$ is rejected (with sufficiently high probability to make it undesirable) as long as $E(c|O) < H-V+\mu\phi(e)-[(H-O)/\lambda(n)]$.

But all partial pooling equilibria fail to be "universally divine", as defined by Banks and Sobel (1987). Suppose that there is a partial pooling equilibrium at O^* , and consider deviations to some price O , where $O > O^*$. Define $\theta(c;O)$ as the probability of rejecting the offer that makes an HMO₀ of level c indifferent between the equilibrium offer O^* and O . By definition, this means $[O^*-c+\phi(e)]=(1-\theta(c;O))[O-c+\phi(e)]$, which can be rewritten as $\theta(c;O)=\frac{[O-O^*]}{O-c+\phi(e)} \in (0, 1)$. Since $\theta'(c;O)=[O-O^*]/[O-c+\phi(e)]^2 > 0$ for $O > O^*$, $\theta(c;O)$ is maximized at $c=c_s$. This implies that c_s is the level most likely to deviate to O . According to Banks and Sobel (1987), universal divinity requires the

customer to believe that an out-of-equilibrium offer only comes from the level most likely to deviate; in this case, it requires the customer to believe that any HMO_0 which offers $O > O^*$ is of level c_x with probability 1. Thus, for $O < H - \lambda(n)[H - V - c_x + \mu\phi(e)]$, the best response of the customer is to accept the offer with probability 1. Then, offer O will be made (i.e., deviation will definitely happen), and therefore, O^* is not universally divine.

(C) From (A) and (B), all pooling equilibria fail to be universally divine.

Therefore, only the separating equilibrium characterized in Proposition 2.1 survives this refinement.

A.2. Proof of Proposition 2.2

Assume the optimal investments I_{nc}^* and e_{nc}^* exist and satisfy the first-order conditions (i.e., equations (2.12) and (2.13)) and the second-order conditions (i.e., $\lambda_{II} < 0$, $\lambda_{ee} < 0$, and $[\lambda_{II}\lambda_{ee} - (\lambda_{Ie})^2] > 0$).

(1) To explore the direction of changes in I_{nc}^* and e_{nc}^* for a change in the parameter n , I totally differentiate equations (2.12) and (2.13) with respect to n and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial n} + \lambda_{Ie} \frac{\partial e}{\partial n} + \lambda_{In} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial n} + \lambda_{ee} \frac{\partial e}{\partial n} + \lambda_{en} = 0 \end{cases}$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial n} = \det \begin{pmatrix} -\lambda_{In} & \lambda_{Ie} \\ -\lambda_{en} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial n} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{In} \\ \lambda_{eI} & -\lambda_{en} \end{pmatrix},$$

where

$$\lambda_{II} = \int_{\underline{c}}^{\bar{c}} F_{II}(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) [1 - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G}] dc < 0,$$

$$\lambda_{Ie} = \int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda(\underline{c}-c)}{G} (1-\lambda\mu) \phi'(e) [(1-\lambda) \frac{\lambda(\underline{c}-c)}{G} - \lambda] dc > 0,$$

$$\lambda_{ee} = \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \left\{ \phi''(e) \left[1 - \frac{\lambda(\underline{c}-c)}{G} \right] + \frac{\lambda^2(\underline{c}-c)^2 (1-\lambda\mu) \phi''(e)}{G^3} \right\} dF(c, I) < 0^2,$$

$$\lambda_{In} = \int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \lambda' \frac{\lambda(\underline{c}-c)}{G} \left\{ 1 - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G} \right\} \frac{H-c+\phi(e)}{G} + 1 \} dc < 0,$$

$$\lambda_{eV} = - \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \lambda' \phi'(e) \left\{ \frac{[H-c+\phi(e)] \lambda(\underline{c}-c)^2 (1-\lambda\mu)}{G^3} + \mu \left(1 - \frac{\lambda(\underline{c}-c)}{G} \right) \right\} dF(c, I) < 0.$$

Thus, $\partial I_{nc}^*/\partial n < 0$, and $\partial e_{nc}^*/\partial n < 0$.

(2) Similarly, to explore the direction of changes in I_{nc}^* and e_{nc}^* for a change in the parameter V , I totally differentiate equations (2.12) and (2.13) with respect to V and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial V} + \lambda_{Ie} \frac{\partial e}{\partial V} + \lambda_{IV} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial V} + \lambda_{ee} \frac{\partial e}{\partial V} + \lambda_{eV} = 0 \end{cases}.$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial V} = \det \begin{pmatrix} -\lambda_{IV} & \lambda_{Ie} \\ -\lambda_{eV} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial V} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{IV} \\ \lambda_{eI} & -\lambda_{eV} \end{pmatrix},$$

where

$$\lambda_{IV} = - \int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda^2(\underline{c}-c)}{G^2} [\lambda - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G}] dc > 0,$$

$$\lambda_{eV} = \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1-\lambda\mu) \phi'(e) \frac{\lambda^3(\underline{c}-c)^2}{G^3} dF(c, I) > 0.$$

Thus, $\partial I_{nc}^*/\partial V > 0$, and $\partial e_{nc}^*/\partial V > 0$.

² λ_{ee} is assumed to be negative so that the second-order conditions of the maximization problem are satisfied. In other words, the curvature of $\phi(e(x))$ is assumed to be large.

(3) Likewise, to explore the direction of changes in I_{nc}^* and e_{nc}^* for a change in the parameter H , I totally differentiate equations (2.12) and (2.13) with respect to H and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial H} + \lambda_{Ie} \frac{\partial e}{\partial H} + \lambda_{IH} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial H} + \lambda_{ee} \frac{\partial e}{\partial H} + \lambda_{eH} = 0 \end{cases}$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial H} = \det \begin{pmatrix} -\lambda_{IH} & \lambda_{Ie} \\ -\lambda_{eH} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial H} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{IH} \\ \lambda_{eI} & -\lambda_{eH} \end{pmatrix},$$

where

$$\lambda_{IH} = -\int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda(1-\lambda)(\underline{c}-c)}{G^2} [\lambda - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G}] dc > 0,$$

$$\lambda_{eH} = \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1-\lambda\mu) \phi'(e) \frac{\lambda^2(1-\lambda)(\underline{c}-c)^2}{G^3} dF(c, I) > 0.$$

Thus, $\partial I_{nc}^*/\partial H > 0$, and $\partial e_{nc}^*/\partial H > 0$.

(4) Similarly, to explore the direction of changes in I_{nc}^* and e_{nc}^* for a change in the parameter μ , I totally differentiate equations (2.12) and (2.13) with respect to μ and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial \mu} + \lambda_{Ie} \frac{\partial e}{\partial \mu} + \lambda_{I\mu} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial \mu} + \lambda_{ee} \frac{\partial e}{\partial \mu} + \lambda_{e\mu} = 0 \end{cases}$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial \mu} = \det \begin{pmatrix} -\lambda_{I\mu} & \lambda_{Ie} \\ -\lambda_{e\mu} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial \mu} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{I\mu} \\ \lambda_{eI} & -\lambda_{e\mu} \end{pmatrix},$$

where

$$\lambda_{I\mu} = \int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda^2(\underline{c}-c)\phi}{G^2} [\lambda - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G}] dc < 0,$$

$$\lambda_{e\mu} = - \int_{\underline{e}}^{\bar{e}} \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) \lambda \phi'(e) \left\{ \frac{\lambda^2(\underline{e}-c)^2(1-\lambda\mu)}{G^3} + \left(1 - \frac{\lambda(\underline{e}-c)}{G}\right) \right\} dF(c, I) < 0.$$

Thus, $\partial I_{nc}^*/\partial \mu < 0$, and $\partial e_{nc}^*/\partial \mu < 0$.

A.3. Proof of Proposition 2.3

Assume the optimal investments I_c^* and e_c^* exist and satisfy the first-order conditions (i.e., equations (17) and (18)) and the second-order conditions (i.e., $\lambda_{II} < 0$, $\lambda_{ee} < 0$, and $[\lambda_{II}\lambda_{ee} - (\lambda_{Ie})^2] > 0$).

(1) To explore the direction of changes in I_c^* and e_c^* for a change in the parameter n , I totally differentiate equations (2.17) and (2.18) with respect to n and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial n} + \lambda_{Ie} \frac{\partial e}{\partial n} + \lambda_{In} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial n} + \lambda_{ee} \frac{\partial e}{\partial n} + \lambda_{en} = 0 \end{cases}.$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial n} = \det \begin{pmatrix} -\lambda_{In} & \lambda_{Ie} \\ -\lambda_{en} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial n} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{In} \\ \lambda_{eI} & -\lambda_{en} \end{pmatrix},$$

where

$$\lambda_{II} = \int_{\underline{e}}^{\bar{e}} F_{II}(c, I) \left\{ \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) [1 - (1-\lambda) \frac{\lambda(\underline{e}-c)}{G}] + \lambda \right\} dc < 0,$$

$$\lambda_{Ie} = \int_{\underline{e}}^{\bar{e}} F_I(c, I) \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) \frac{\lambda(\underline{e}-c)}{G} (1-\lambda\mu) \phi'(e) [(1-\lambda) \frac{\lambda(\underline{e}-c)}{G} - \lambda] dc > 0,$$

$$\lambda_{ee} = \int_{\underline{e}}^{\bar{e}} \left\{ \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) \phi''(e) \left[1 - \frac{\lambda(\underline{e}-c)}{G}\right] + \frac{\lambda^2(\underline{e}-c)^2(1-\lambda\mu)\phi'(\underline{e})^2}{G^3} \right\} + \lambda\mu\phi''(e) dF(c, I) < 0,$$

$$\lambda_{In} = \int_{\underline{e}}^{\bar{e}} F_I(c, I) \lambda' \left\{ \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) \frac{\lambda(\underline{e}-c)}{G} \left[1 - (1-\lambda) \frac{\lambda(\underline{e}-c)}{G}\right] \frac{H-c+\theta(e)}{G} + 1 \right\} + 1 \right\} dc,$$

$$\lambda_{eI} = - \int_{\underline{e}}^{\bar{e}} \exp\left(\frac{\lambda(\underline{e}-c)}{G}\right) \lambda' \phi' \left\{ \frac{H-c+\theta(e)\lambda(\underline{e}-c)^2(1-\lambda\mu)}{G^3} + \mu \left(1 - \frac{\lambda(\underline{e}-c)}{G}\right) \right\} dF(c, I) < 0.$$

If $\lambda_{In} < 0$, I_c^* and e_c^* decrease with n , the same as in the non-commitment case. However, if $\lambda_{In} > 0$, $\partial I_c^*/\partial n$ and $\partial e_c^*/\partial n$ are indefinite.

(2) Similarly, to explore the direction of changes in I_c^* and e_c^* for a change in the parameter V , I totally differentiate equations (2.17) and (2.18) with respect to V and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial V} + \lambda_{Ie} \frac{\partial e}{\partial V} + \lambda_{IV} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial V} + \lambda_{ee} \frac{\partial e}{\partial V} + \lambda_{eV} = 0 \end{cases}$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial V} = \det \begin{pmatrix} -\lambda_{IV} & \lambda_{Ie} \\ -\lambda_{eV} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial V} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{IV} \\ \lambda_{eI} & -\lambda_{eV} \end{pmatrix},$$

where

$$\lambda_{IV} = - \int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(c-c)}{G}\right) \frac{\lambda^2(c-c)}{G^2} [\lambda - (1-\lambda) \frac{\lambda(c-c)}{G}] dc > 0,$$

$$\lambda_{eV} = \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) (1 - \lambda\mu) \phi'(e) \frac{\lambda^3(c-c)^2}{G^3} dF(c, I) > 0.$$

Thus, $\partial I_c^*/\partial V > 0$, and $\partial e_c^*/\partial V > 0$, the same as in the non-commitment case.

(3) Likewise, to explore the direction of changes in I_c^* and e_c^* for a change in the parameter H , I totally differentiate equations (2.17) and (2.18) with respect to H and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial H} + \lambda_{Ie} \frac{\partial e}{\partial H} + \lambda_{IH} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial H} + \lambda_{ee} \frac{\partial e}{\partial H} + \lambda_{eH} = 0 \end{cases}$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial H} = \det \begin{pmatrix} -\lambda_{IH} & \lambda_{Ie} \\ -\lambda_{eH} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial H} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{IH} \\ \lambda_{eI} & -\lambda_{eH} \end{pmatrix},$$

where

$$\lambda_{IH} = -\int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda(1-\lambda)(\underline{c}-c)}{G^2} [\lambda - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G}] dc > 0,$$

$$\lambda_{eH} = \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1-\lambda\mu) \phi'(c) \frac{\lambda^2(1-\lambda)(\underline{c}-c)^2}{G^3} dF(c, I) > 0.$$

Thus, $\partial I_c^*/\partial H > 0$, and $\partial e_c^*/\partial H > 0$, the same as in the non-commitment case.

(4) Similarly, to explore the direction of changes in I_c^* and e_c^* for a change in the parameter μ , I totally differentiate equations (2.17) and (2.18) with respect to μ and get

$$\begin{cases} \lambda_{II} \frac{\partial I}{\partial \mu} + \lambda_{Ie} \frac{\partial e}{\partial \mu} + \lambda_{I\mu} = 0 \\ \lambda_{eI} \frac{\partial I}{\partial \mu} + \lambda_{ee} \frac{\partial e}{\partial \mu} + \lambda_{e\mu} = 0 \end{cases}.$$

Applying Cramer's rule yields

$$\frac{\partial I^*}{\partial \mu} = \det \begin{pmatrix} -\lambda_{I\mu} & \lambda_{Ie} \\ -\lambda_{e\mu} & \lambda_{ee} \end{pmatrix}, \text{ and } \frac{\partial e^*}{\partial \mu} = \det \begin{pmatrix} \lambda_{II} & -\lambda_{I\mu} \\ \lambda_{eI} & -\lambda_{e\mu} \end{pmatrix},$$

where

$$\lambda_{I\mu} = \int_{\underline{c}}^{\bar{c}} F_I(c, I) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda^2(\underline{c}-c)}{G^2} [\lambda - (1-\lambda) \frac{\lambda(\underline{c}-c)}{G}] dc < 0,$$

$$\lambda_{e\mu} = \int_{\underline{c}}^{\bar{c}} \lambda \phi'(c) \left\{ -\exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \left(1 - \frac{\lambda(\underline{c}-c)}{G}\right) - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda^2(\underline{c}-c)^2(1-\lambda\mu)}{G^3} \right\} dF(c, I).$$

If $\lambda_{e\mu} < 0$, I_c^* and e_c^* decrease with μ , the same as in the non-commitment case. However,

if $\lambda_{e\mu} > 0$, $\partial I_c^*/\partial \mu$ and $\partial e_c^*/\partial \mu$ are indefinite.

A.4. Proof of Proposition 2.4

Recall that I_{nc}^* and e_{nc}^* satisfy equations (2.12) and (2.13), I_c^* and e_c^* satisfy equations (2.17) and (2.18), and I^{**} and e^{**} satisfy equations (2.19) and (2.20).

(1) Since $0 < \exp(\frac{\lambda(\underline{c}-c)}{G})(1 - \frac{\lambda(\underline{c}-c)}{G}) \leq 1$, $\lambda\mu < w \equiv (1 - \lambda\mu) \exp(\frac{\lambda(\underline{c}-c)}{G})(1 - \frac{\lambda(\underline{c}-c)}{G}) + \lambda\mu \leq 1$.

Then, $T_{nc} \equiv \int_{\underline{c}}^{\bar{c}} (1 - \lambda\mu) \exp(\frac{\lambda(\underline{c}-c)}{G})(1 - \frac{\lambda(\underline{c}-c)}{G}) dF < T_c \equiv \int_{\underline{c}}^{\bar{c}} w dF < 1$. Now, $\phi'(e_{nc}^*) = 1/T_{nc} >$

$\phi'(e_c^*) = 1/T_c > \phi'(e^{**}) = 1$, and $\phi''(e) < 0$. Hence, $e_{nc}^* < e_c^* < e^{**}$.

(2) It is clear that $I_{nc}^* < I_c^*$, because $\{\exp(\frac{\lambda(\underline{c}-c)}{G})[1 - (1 - \lambda)\frac{\lambda(\underline{c}-c)}{G}]\} < \{\exp(\frac{\lambda(\underline{c}-c)}{G})[1 - (1 - \lambda)\frac{\lambda(\underline{c}-c)}{G}] + \lambda\}$ and $F_{II}(c, I) < 0$.

(3) Since $\exp((1 - \lambda)\frac{\lambda(\underline{c}-c)}{G})[1 - (1 - \lambda)\frac{\lambda(\underline{c}-c)}{G}] \in (0, 1]$ and $\exp(\frac{\lambda(\underline{c}-c)}{G}) < \exp((1 - \lambda)\frac{\lambda(\underline{c}-c)}{G})$, I obtain $\exp(\frac{\lambda(\underline{c}-c)}{G})[1 - (1 - \lambda)\frac{\lambda(\underline{c}-c)}{G}] < 1$. Thus, combined with $F_{II}(c, I) < 0$, $I_{nc}^* < I^{**}$.

(4) Note that $Y(c) \equiv \{\exp(\frac{\lambda(\underline{c}-c)}{G})[1 - (1 - \lambda)\frac{\lambda(\underline{c}-c)}{G}] + \lambda\}$ decreases with c and $Y(\underline{c}) = 1 + \lambda > 1$.

If $Y(\bar{c}) > 1$, $Y(c) > 1$ for all $c \in [\underline{c}, \bar{c}]$. Thus, combined with $F_{II}(c, I) < 0$, $I_c^* > I^{**}$.

A.5. Proof of Proposition 2.5

First, it can be checked that $dP_{nc}/dI_{nc} + dP_{nc}/de_{nc} > 0$. To see this,

$$\begin{aligned} \frac{\partial P_{nc}}{\partial I_{nc}} &= \frac{1 - \lambda}{n - 1} \int_{\underline{c}}^{\bar{c}} [H - V - c + \mu\phi(e)] \left[1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right] dF_I > 0, \text{ and} \\ \frac{\partial P_{nc}}{\partial e_{nc}} &= \frac{1 - \lambda}{n - 1} \phi' \int_{\underline{c}}^{\bar{c}} \left\{ \mu(1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) + [H - V - c + \mu\phi(e)] \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda(\underline{c}-c)}{G^2} (1 - \lambda\mu) \right\} dF \\ &= \frac{1 - \lambda}{n - 1} \phi' \int_{\underline{c}}^{\bar{c}} \left\{ \mu(1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right)) + \frac{[H - c + \phi(e)] - G}{\lambda} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda(\underline{c}-c)}{G^2} (1 - \lambda\mu) \right\} dF \\ &= \frac{1 - \lambda}{n - 1} \int_{\underline{c}}^{\bar{c}} \left\{ \phi' \mu - \mu \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \phi' + \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1 - \lambda\mu) \frac{H - (c + \phi(e))}{G} \frac{\lambda(\underline{c}-c)}{G} - \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1 - \lambda\mu) \frac{\lambda(\underline{c}-c)}{G} \right\} dF \\ &= \frac{1 - \lambda}{n - 1} \left\{ \int_{\underline{c}}^{\bar{c}} \left\{ \phi' \mu + \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1 - \lambda\mu) \frac{H - (c + \phi(e))}{G} \frac{\lambda(\underline{c}-c)}{G} - \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right\} dF + \frac{1}{\lambda} \right\} \end{aligned}$$

$$\begin{aligned}
& (\because \int_{\epsilon}^{\infty} \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) (1-\lambda\mu)\phi'(1-\frac{\lambda(\epsilon-c)}{G}) dF = 1) \\
& > \frac{1}{n-1} \left\{ (1-\lambda)\phi'\mu - \frac{1-\lambda}{\lambda} \int_{\epsilon}^{\infty} \phi' \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) dF + \int_{\epsilon}^{\infty} \phi' \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(\epsilon-c)}{G} dF + \frac{1-\lambda}{\lambda} \right\} \\
& \quad (\because 0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\theta(e))}{G} < 1) \\
& = \frac{1}{n-1} \left\{ \phi'\mu + [(1-\lambda\mu) - \frac{1-\lambda}{\lambda}] \phi' \int_{\epsilon}^{\infty} \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) dF + \frac{1-2\lambda}{\lambda} \right\} \\
& > \frac{1-\lambda}{n-1} \mu \frac{1}{1-\lambda\mu} + \frac{1}{n-1} \frac{2\lambda-1-\lambda^2\mu}{\lambda} \frac{1}{1-\lambda\mu} + \frac{1}{n-1} \frac{1-2\lambda}{\lambda} = 0. \\
& (\because \phi' \int_{\epsilon}^{\infty} \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) dF = \frac{\int_{\epsilon}^{\infty} \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) dF}{(1-\lambda\mu) \int_{\epsilon}^{\infty} \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) [1-\frac{\lambda(\epsilon-c)}{G}] dF} < \frac{1}{1-\lambda\mu}, \text{ and } 2\lambda-1-\lambda^2\mu < 0.)
\end{aligned}$$

Second, let $I_{nc}=I_c^*$, $e_{nc}=e_c^*$, and $P_{nc}=P_c^*$. Proposition 2.4 implies $I_{nc}^* < I_{nc}$ and $e_{nc}^* < e_{nc}$, which, combined with the above result, yields $P_{nc}^* < P_{nc}=P_c^*$. Thus, $P_{nc}^* < P_c^*$.

A.6. Proof of Lemma 2.1

Notice that $\frac{d\Pi_n}{dn} = \frac{\partial\Pi_n}{\partial I} \frac{dI}{dn} + \frac{\partial\Pi_n}{\partial e} \frac{de}{dn} + \frac{\partial\Pi_n}{\partial n}$.

(1) $\partial\Pi_n/\partial I > 0$ by assumption.

(2) $\partial\Pi_n/\partial e > 0$. To see this,

$$\begin{aligned}
\frac{\partial\Pi_n}{\partial e} &= \frac{1-\lambda}{n-1} \phi' \int_{\epsilon}^{\infty} \left\{ \mu(1-\exp\left(\frac{\lambda(\epsilon-c)}{G}\right)) + [H-V-c+\mu\phi(e)] \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) \frac{\lambda(\epsilon-c)}{G^2} (1-\lambda\mu) \right\} dF \\
&= \frac{1-\lambda}{n-1} \phi' \int_{\epsilon}^{\infty} \left\{ \mu(1-\exp\left(\frac{\lambda(\epsilon-c)}{G}\right)) + \frac{[H-c+\theta(e)]-G}{\lambda} \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) \frac{\lambda(\epsilon-c)}{G^2} (1-\lambda\mu) \right\} dF \\
&= \frac{1-\lambda}{n-1} \int_{\epsilon}^{\infty} \left\{ \phi'\mu - \mu \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) \phi' + \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) (1-\lambda\mu) \frac{H-(c-\theta(e))}{G} \frac{\lambda(\epsilon-c)}{G} - \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\epsilon-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(\epsilon-c)}{G} \right\} dF
\end{aligned}$$

$$\begin{aligned}
&= \frac{1-\lambda}{n-1} \left\{ \int_{\underline{c}}^{\bar{c}} \left[\phi' \mu + \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(\underline{c}-c)}{G} \frac{[H-c+\phi(e)]}{\lambda} - \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right] dF + \frac{1}{\lambda} \right\} \\
&\quad \left(\because \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1-\lambda\mu) \phi' (1 - \left(\frac{\lambda(\underline{c}-c)}{G}\right)) dF = 1 \right) \\
&> \frac{1}{n-1} \left\{ (1-\lambda) \phi' \mu - \frac{1-\lambda}{\lambda} \int_{\underline{c}}^{\bar{c}} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) dF + \int_{\underline{c}}^{\bar{c}} \phi' \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(\underline{c}-c)}{G} dF + \frac{1-\lambda}{\lambda} \right\} \\
&\quad \left(\because 0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} < 1 \right) \\
&= \frac{1}{n-1} \left\{ \phi' \mu + \left[(1-\lambda\mu) - \frac{1-\lambda}{\lambda} \right] \phi' \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) dF + \frac{1-2\lambda}{\lambda} \right\} \\
&> \frac{1-\lambda}{n-1} \mu \frac{1}{1-\lambda\mu} + \frac{1}{n-1} \frac{2\lambda-1-\lambda^2\mu}{\lambda} \frac{1}{1-\lambda\mu} + \frac{1}{n-1} \frac{1-2\lambda}{\lambda} = 0. \\
&\quad \left(\because \phi' \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) dF = \frac{\int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) dF}{(1-\lambda\mu) \int_{\underline{c}}^{\bar{c}} \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \left[1 - \frac{\lambda(\underline{c}-c)}{G} \right] dF} < \frac{1}{1-\lambda\mu}, \text{ and } 2\lambda-1-\lambda^2\mu < 0. \right)
\end{aligned}$$

(3) $\partial \Pi_n / \partial n < 0$. To see this, note that

$$\begin{aligned}
\frac{\partial \Pi_n}{\partial n} &= -\frac{2}{n^3} - \frac{1-\lambda}{(n-1)^2} \int_{\underline{c}}^{\bar{c}} [H-V-c+\mu\phi(e)] \left[1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right] dF \\
&\quad - \frac{\lambda'}{n-1} \int_{\underline{c}}^{\bar{c}} [H-V-c+\mu\phi(e)] \left[1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right] dF + \frac{1-\lambda}{n-1} \frac{\partial M}{\partial n},
\end{aligned}$$

$$\begin{aligned}
\text{where } \frac{\partial M}{\partial n} &= \int_{\underline{c}}^{\bar{c}} [H-V-c+\mu\phi(e)] \left(-\exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right) \frac{\lambda G + \lambda \lambda' [H-V-c+\mu\phi(e)]}{G^2} (\underline{c}-c) dF \\
&= - \int_{\underline{c}}^{\bar{c}} [H-V-c+\mu\phi(e)] \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{\lambda'(\underline{c}-c) [H-c-\phi(e)]}{G^2} dF.
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \frac{1-\lambda}{n-1} \frac{\partial M}{\partial n} - \frac{\lambda'}{n-1} \int_{\underline{c}}^{\bar{c}} [H-V-c+\mu\phi(e)] \left[1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right] dF \\
&= \frac{-\lambda'}{n-1} \int_{\underline{c}}^{\bar{c}} [H-V-c+\mu\phi(e)] \lambda' \left\{ (1-\lambda) \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \frac{(\underline{c}-c) [H-c-\phi(e)]}{G^2} + 1 - \exp\left(\frac{\lambda(\underline{c}-c)}{G}\right) \right\} dF \leq 0,
\end{aligned}$$

$$\begin{aligned}
(\because \text{the bracket} &= \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \frac{\lambda(\varepsilon-c)}{G} \frac{(1-\lambda)}{\lambda} \frac{[H-c-\theta(e)]}{G} + 1 - \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \\
&> \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \frac{\lambda(\varepsilon-c)}{G} + 1 - \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \\
&= 1 - \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \left[1 - \frac{\lambda(\varepsilon-c)}{G}\right] \geq 0.)
\end{aligned}$$

Since all other terms in $\partial\Pi_n/\partial n$ are negative, combined with the above result, $\partial\Pi_n/\partial n < 0$.

From (1), (2), (3), and results in Proposition 2.2, I obtain $d\Pi_n/dn < 0$.

A.7. Proof of Lemma 2.2

Recall the social welfare function given by equation (2.23), the overall expected profit of an HMO given by equation (2.22), and the expression of $\partial\Pi_n/\partial n$ obtained in the proof of Lemma 2.1.

$$\text{Notice that } SW'(n) = \frac{\partial SW}{\partial I} \frac{dI}{dn} + \frac{\partial SW}{\partial e} \frac{de}{dn} + \left(\frac{\partial SW}{\partial n} - \Pi_n \right) + \Pi_n.$$

(1) $\partial SW/\partial I > 0$. To see this,

$$\frac{\partial SW}{\partial I} = (n-1) \frac{\partial \Pi_n}{\partial I} + \int_{\underline{e}}^{\bar{e}} F_I \lambda dc > 0.$$

(2) $\partial SW/\partial e > 0$. To see this,

$$\begin{aligned}
\frac{\partial SW}{\partial e} &= \int_{\underline{e}}^{\bar{e}} \left\{ -\frac{1-\lambda}{n-1} \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \frac{\lambda(\varepsilon-c)(1-\lambda)\mu'}{G^2} \right\} [H-V-c+\mu\phi(e)] + \left[\lambda - \frac{1-\lambda}{n-1} \left(1 - \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \right) \mu\phi' \right] dF \\
&\quad + n \frac{1-\lambda}{n-1} \phi' \int_{\underline{e}}^{\bar{e}} \left\{ \mu \left(1 - \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \right) + [H-V-c+\mu\phi(e)] \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \frac{\lambda(\varepsilon-c)(1-\lambda)\mu}{G^2} \right\} dF \\
&= \lambda\mu\phi' + (1-\lambda) \int_{\underline{e}}^{\bar{e}} \phi' \left\{ \mu \left(1 - \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \right) + [H-V-c+\mu\phi(e)] \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) \frac{\lambda(\varepsilon-c)(1-\lambda)\mu}{G^2} \right\} dF \\
&= \lambda\mu\phi' + (1-\lambda) \phi' \int_{\underline{e}}^{\bar{e}} \left\{ \mu - \frac{1}{\lambda} \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) + \frac{1}{\lambda} \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(\varepsilon-c)}{G} \frac{H-c-\theta(e)}{G} \right\} dF + \frac{1-\lambda}{\lambda} \\
&(\because \int_{\underline{e}}^{\bar{e}} \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) (1-\lambda\mu) \phi' dF = 1, \text{ and } [H-V-c+\mu\phi(e)] = \frac{[H-c-\theta(e)]-G}{\lambda}) \\
&> \mu\phi' - \frac{1-\lambda}{\lambda} \phi' \int_{\underline{e}}^{\bar{e}} \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) dF + \phi' \int_{\underline{e}}^{\bar{e}} \exp\left(\frac{\lambda(\varepsilon-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(\varepsilon-c)}{G} dF + \frac{1-\lambda}{\lambda}
\end{aligned}$$

$$(\because 0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\theta(e))}{G} < 1)$$

$$= \mu\phi' + [(1-\lambda\mu) - \frac{1-\lambda}{\lambda}]\phi' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF + \frac{1-2\lambda}{\lambda}$$

$$> \mu \frac{1}{1-\lambda\mu} + \frac{2\lambda-1-\lambda^2\mu}{\lambda} \frac{1}{1-\lambda\mu} + \frac{1-2\lambda}{\lambda} = \frac{\lambda\mu}{1-\lambda\mu} > 0. \quad (\because (a) 2\lambda - 1 - \lambda^2\mu < 0,$$

$$(b) \phi' > \frac{1}{1-\lambda\mu}, \text{ as } \phi'(1-\lambda\mu) \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) \left(1 - \left(\frac{\lambda(c-c)}{G}\right)\right) dF = 1 \text{ and } \exp\left(\frac{\lambda(c-c)}{G}\right) \left(1 - \left(\frac{\lambda(c-c)}{G}\right)\right)$$

$$\in (0, 1]; \text{ and (c) } \phi' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF = \frac{\int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF}{(1-\lambda\mu) \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) \left[1 - \frac{\lambda(c-c)}{G}\right] dF} < \frac{1}{1-\lambda\mu}.)$$

(3) $\partial SW / \partial n - \Pi_n < 0$. To see this,

$$\begin{aligned} \frac{\partial SW}{\partial n} = & \int_c^{\bar{c}} \left\{ \exp\left(\frac{\lambda(c-c)}{G}\right) G(c-c) \frac{\lambda'G + \lambda\lambda'[H-V-c+\mu\phi(e)]}{G^2} - \exp\left(\frac{\lambda(c-c)}{G}\right) \lambda' [H-V-c+\mu\phi(e)] \right\} dF \\ & + \int_c^{\bar{c}} \lambda [H-V-c+\mu\phi(e)] dF + \int_c^{\bar{c}} \frac{1-\lambda}{n^2} \left(1 - \exp\left(\frac{\lambda(c-c)}{G}\right)\right) [H-V-c+\mu\phi(e)] dF \\ & + \frac{\lambda'}{n-1} \int_c^{\bar{c}} \left(1 - \exp\left(\frac{\lambda(c-c)}{G}\right)\right) [H-V-c+\mu\phi(e)] dF \\ & + \int_c^{\bar{c}} \frac{1-\lambda}{n-1} \exp\left(\frac{\lambda(c-c)}{G}\right) (c-c) \frac{\lambda'G + \lambda\lambda'[H-V-c+\mu\phi(e)]}{G^2} [H-V-c+\mu\phi(e)] dF \\ & + \frac{5}{4} \frac{1}{n^3} - \frac{2}{n^2} - \frac{n(1-\lambda)}{(n-1)^2} \int_c^{\bar{c}} [H-V-c+\mu\phi(e)] \left(1 - \exp\left(\frac{\lambda(c-c)}{G}\right)\right) dF \\ & - \frac{n\lambda'}{n-1} \int_c^{\bar{c}} [H-V-c+\mu\phi(e)] \left(1 - \exp\left(\frac{\lambda(c-c)}{G}\right)\right) dF \\ & - \frac{n(1-\lambda)}{n-1} \int_c^{\bar{c}} [H-V-c+\mu\phi(e)] \exp\left(\frac{\lambda(c-c)}{G}\right) \frac{\lambda'(c-c)[H-V-c+\mu\phi(e)]}{G^2} dF + \Pi_n. \end{aligned}$$

Thus, after simplifying,

$$\begin{aligned} \frac{\partial SW}{\partial n} - \Pi_n = & -\frac{3}{4} \frac{1}{n^3} - \frac{1-\lambda}{n-1} \int_c^{\bar{c}} [H-V-c+\mu\phi(e)] \left(1 - \exp\left(\frac{\lambda(c-c)}{G}\right)\right) dF \\ & + \lambda' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) \frac{(c-c)[H-V-c+\mu\phi(e)]}{G^2} [V + (1-\mu)\phi] dF < 0. \end{aligned}$$

From (1), (2), (3), and results in Proposition 2.2, I obtain $SW'(n) < \Pi_n$.

A.8. Proof of Proposition 2.6 (Proposition 2.7)³

From Lemma 2.2 (Lemma 2.4), I have $SW'(n) < \Pi_n$. Let n^* be the number of HMOs in the social optimum, and n^e be the equilibrium number of HMOs in the free-entry market. Then, $0 = SW'(n^*) < \Pi_{n^*}$, as $0 = SW'(n^*)$ by definition. Thus, $\Pi_{n^*} > 0$. Moreover, $\Pi_{n^*} = 0$ by definition. Hence, $0 = \Pi_{n^*} < \Pi_{n^*}$. Since $d\Pi_n/dn < 0$, I obtain the result that $n^e > n^*$.

A.9. Proof of Lemma 2.3

Notice that $\frac{d\Pi_n}{dn} = \frac{\partial\Pi_n}{\partial l} \frac{dl}{dn} + \frac{\partial\Pi_n}{\partial e} \frac{de}{dn} + \frac{\partial\Pi_n}{\partial n}$.

(1) $\partial\Pi_n/\partial l > 0$ by assumption.

(2) $\partial\Pi_n/\partial e > 0$. To see this,

$$\begin{aligned}
 \frac{\partial\Pi_n}{\partial e} &= \frac{1-\lambda}{n-1} \phi' \int_{\bar{e}}^{\bar{c}} \left\{ \mu \left(1 - \exp\left(\frac{\lambda(c-e)}{G}\right) \right) + [H - V - c + \mu\phi(e)] \exp\left(\frac{\lambda(c-e)}{G}\right) \frac{\lambda(c-e)}{G^2} (1 - \lambda\mu) \right\} dF \\
 &= \frac{1-\lambda}{n-1} \phi' \int_{\bar{e}}^{\bar{c}} \left\{ \mu \left(1 - \exp\left(\frac{\lambda(c-e)}{G}\right) \right) + \frac{[H - c + \phi(e)] - G}{\lambda} \exp\left(\frac{\lambda(c-e)}{G}\right) \frac{\lambda(c-e)}{G^2} (1 - \lambda\mu) \right\} dF \\
 &= \frac{1-\lambda}{n-1} \int_{\bar{e}}^{\bar{c}} \left\{ \phi' \mu - \mu \exp\left(\frac{\lambda(c-e)}{G}\right) \phi' + \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(c-e)}{G}\right) (1 - \lambda\mu) \frac{H - c + \phi(e)}{G} \frac{\lambda(c-e)}{G} - \frac{1}{\lambda} \phi' \exp\left(\frac{\lambda(c-e)}{G}\right) (1 - \lambda\mu) \frac{\lambda(c-e)}{G} \right\} dF \\
 &= \frac{1-\lambda}{n-1} \left\{ \int_{\bar{e}}^{\bar{c}} \left\{ \phi' \frac{1}{\lambda} \exp\left(\frac{\lambda(c-e)}{G}\right) \left[-1 + (1 - \lambda\mu) \frac{\lambda(c-e)}{G} \frac{[H - c + \phi(e)]}{\lambda} \right] \right\} dF + \frac{1}{\lambda} \right\} \\
 &(\because \int_{\bar{e}}^{\bar{c}} \left\{ \exp\left(\frac{\lambda(c-e)}{G}\right) (1 - \lambda\mu) \phi' \left(1 - \frac{\lambda(c-e)}{G} \right) + \lambda\mu \phi' \right\} dF = 1.) \\
 &> \frac{1}{n-1} \left\{ -\frac{1-\lambda}{\lambda} \int_{\bar{e}}^{\bar{c}} \phi' \exp\left(\frac{\lambda(c-e)}{G}\right) dF + \int_{\bar{e}}^{\bar{c}} \phi' \exp\left(\frac{\lambda(c-e)}{G}\right) (1 - \lambda\mu) \frac{\lambda(c-e)}{G} dF + \frac{1-\lambda}{\lambda} \right\}
 \end{aligned}$$

³ The technique used for this proof is similar to that used by Mankiw and Whinston (1986).

$$(\because 0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\theta(e))}{G} < 1)$$

$$= \frac{1}{n-1} \left\{ \lambda \mu \phi' + \left[(1-\lambda\mu) - \frac{1-\lambda}{\lambda} \right] \phi' \int_c^{\infty} \exp\left(\frac{\lambda(c-e)}{G}\right) dF + \frac{1-2\lambda}{\lambda} \right\}$$

$$> \frac{1}{n-1} \left\{ \lambda \mu \cdot 1 + \frac{2\lambda - 1 - \lambda^2 \mu}{\lambda} \cdot 1 + \frac{1-2\lambda}{\lambda} \right\} = 0.$$

$$(\because (a) \phi' \int_c^{\infty} \exp\left(\frac{\lambda(c-e)}{G}\right) dF = \frac{\int_c^{\infty} \exp\left(\frac{\lambda(c-e)}{G}\right) dF}{(1-\lambda\mu) \int_c^{\infty} \exp\left(\frac{\lambda(c-e)}{G}\right) \left[1 - \frac{\lambda(c-e)}{G} \right] dF + \lambda \mu} < 1; (b) \phi'(e) > 1; \text{ and } (c)$$

$$2\lambda - 1 - \lambda^2 \mu < 0.)$$

(3) $\partial \Pi_n / \partial n < 0$, as shown in the proof of Lemma 2.1.

From (1), (2), (3), and conditions of $dI/dn < 0$ and $de/dn < 0$, I obtain $d\Pi_n/dn < 0$.

A.10. Proof of Lemma 2.4

Recall the social welfare function given by equation (2.26), the overall expected profit of an HMO given by equation (2.25), and the expression of $\partial \Pi_n / \partial n$ obtained in the proof of Lemma 2.1.

$$\text{Notice that } SW'(n) = \frac{\partial SW}{\partial I} \frac{dI}{dn} + \frac{\partial SW}{\partial e} \frac{de}{dn} + \left(\frac{\partial SW}{\partial n} - \Pi_n \right) + \Pi_n.$$

(1) $\partial SW / \partial I > 0$. To see this,

$$\frac{\partial SW}{\partial I} = (n-1) \frac{\partial \Pi_n}{\partial I} > 0.$$

(2) $\partial SW / \partial e > 0$. To see this,

$$\begin{aligned} \frac{\partial SW}{\partial e} &= (1-\lambda) \int_c^{\infty} \mu \left[1 - \exp\left(\frac{\lambda(c-e)}{G}\right) \right] + [H - V - c + \mu \phi(e)] \exp\left(\frac{\lambda(c-e)}{G}\right) \frac{\lambda(c-e)(1-\lambda\mu)}{G^2} dF \\ &= -\frac{1-\lambda}{\lambda} \phi' \int_c^{\infty} \exp\left(\frac{\lambda(c-e)}{G}\right) dF + \int_c^{\infty} \frac{1-\lambda}{\lambda} \phi' \exp\left(\frac{\lambda(c-e)}{G}\right) \left(1 - \lambda\mu \right) \frac{\lambda(c-e)}{G} \frac{H-(c-\theta(e))}{G} dF + \frac{1-\lambda}{\lambda} \end{aligned}$$

$$(\because \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) (1-\lambda\mu)\phi'(1-\left(\frac{\lambda(c-c)}{G}\right)) dF = 1-\lambda\mu\phi', [H-V-c+\mu\phi(e)] = \frac{[H-c+\phi(e)]-G}{\lambda}.)$$

$$> -\frac{1-\lambda}{\lambda} \phi' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF + \phi' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) (1-\lambda\mu) \frac{\lambda(c-c)}{G} dF + \frac{1-\lambda}{\lambda}$$

$$(\because 0 < \frac{1-\lambda}{\lambda} \frac{H-(c-\phi(e))}{G} < 1)$$

$$= \lambda\mu\phi' + [(1-\lambda\mu) - \frac{1-\lambda}{\lambda}] \phi' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF + \frac{1-2\lambda}{\lambda}$$

$$> \mu \frac{1}{1-\lambda\mu} + \frac{2\lambda-1-\lambda^2\mu}{\lambda} \frac{1}{1-\lambda\mu} + \frac{1-2\lambda}{\lambda} = \frac{\lambda\mu}{1-\lambda\mu} > 0.$$

$$(\because (a) 2\lambda-1-\lambda^2\mu < 0; (b) \phi' > 1, \text{ as } \phi' \{ (1-\lambda\mu) \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) (1-\left(\frac{\lambda(c-c)}{G}\right)) dF + \lambda\mu \} = 1$$

$$\text{and } \exp\left(\frac{\lambda(c-c)}{G}\right) (1-\left(\frac{\lambda(c-c)}{G}\right)) \in (0, 1]; \text{ and}$$

$$(c) \phi' \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF = \frac{\int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) dF}{(1-\lambda\mu) \int_c^{\bar{c}} \exp\left(\frac{\lambda(c-c)}{G}\right) \left[1 - \frac{\lambda(c-c)}{G}\right] dF + \lambda\mu} < 1.)$$

(3) $\partial SW / \partial n - \Pi_n < 0$, as shown in the proof of Lemma 2.2.

From (1), (2), (3), and conditions of $dl/dn < 0$ and $de/dn < 0$, I obtain $SW'(n) < \Pi_n$.

APPENDIX B PROOFS OF THE MAIN RESULTS IN CHAPTER 3

B.1. Proof of Lemma 3.1

The Nash equilibrium of the complete-information game is found by the intersection of the two reaction functions of the firms. Since the information is complete, the consumer gives consistent beliefs in quality to each good. Hence, the possible price set by the high-quality firm, P_H , is in the region of $[C_H, \pi_H r]$. Similarly, $P_L \in [C_L, \pi_L r]$.

First, for firm H to get a nonzero demand, it must be $\pi_H r - P_H \geq \pi_L r - P_L$ and $\pi_H r - P_H \geq 0$. That is, $P_H \leq (\pi_H - \pi_L)r + P_L$ and $P_H \leq \pi_H r$. The latter condition is nonbinding for $P_L \in [C_L, \pi_L r]$. To maximize its profit, firm H can choose $P_H(P_L) = (\pi_H - \pi_L)r + P_L$ so that it shares the market demand evenly with firm L and makes a profit $\Pi_H(P_L) = [(\pi_H - \pi_L)r + P_L - C_H]/2$. Or, it can choose $P_H(P_L) = (\pi_H - \pi_L)r + P_L - \varepsilon$ with an arbitrary $\varepsilon \in (0, [(\pi_H - \pi_L)r + P_L - C_H]/2)$ so that it gets the entire market and makes a profit $\Pi_H(P_L) = [(\pi_H - \pi_L)r + P_L - C_H - \varepsilon]$. The latter profit is bigger than the former, thus firm H's reaction function is $P_H(P_L) = (\pi_H - \pi_L)r + P_L - \varepsilon, \forall P_L \in [C_L, \pi_L r]$.

Second, for firm L to get a nonzero demand, it must be $\pi_H r - P_H \leq \pi_L r - P_L$ and $0 \leq \pi_L r - P_L$. That is, $P_L \leq P_H - (\pi_H - \pi_L)r$ and $P_L \leq \pi_L r$. The latter condition is nonbinding for $P_H \in [C_H, \pi_H r]$. To maximize its profit, firm L can choose $P_L(P_H) = P_H - (\pi_H - \pi_L)r$ so that it shares the market demand evenly with firm H and makes a profit $\Pi_L(P_H) = [P_H - (\pi_H - \pi_L)r - C_L]/2$. Or, it can choose $P_L(P_H) = P_H - (\pi_H - \pi_L)r - \varepsilon$ with an arbitrary $\varepsilon \in (0, [P_H - (\pi_H - \pi_L)r - C_L]/2)$ so that it

gets the entire market and makes a profit $\Pi_L(P_H)=[P_H-(\pi_H-\pi_L)r-C_L-\varepsilon]$. The latter profit is bigger than the former if ε satisfies the condition. Thus, if $P_H > (\pi_H-\pi_L)r+C_L$, firm L chooses $P_L(P_H)=P_H-(\pi_H-\pi_L)r-\varepsilon$. If $P_H=(\pi_H-\pi_L)r+C_L$, it chooses $P_L(P_H)=P_H-(\pi_H-\pi_L)r=C_L$. If $P_H < (\pi_H-\pi_L)r+C_L$, there is no way for firm L to make a positive profit and it chooses any $P_L(P_H) \in [C_L, \pi_L r]$. Therefore, firm L's reaction function is as follows:

$$P_L(P_H) = \begin{cases} P_H - (\pi_H - \pi_L)r - \varepsilon & \text{if } P_H > (\pi_H - \pi_L)r + C_L \\ C_L & \text{if } P_H = (\pi_H - \pi_L)r + C_L \\ [C_L, Lr] & \text{if } P_H < (\pi_H - \pi_L)r + C_L \end{cases}$$

Finally, combine these two reaction functions. From Figure 3.2, the unique Nash equilibrium is $P_H^*=C_L+(\pi_H-\pi_L)r-\varepsilon$ and $P_L^*=C_L$.

B.2. Proof of Proposition 3.1

The calculation is straightforward. Thus, I only summarize the results for the three cases other than the case in Lemma 3.1.

(1) $(R_H, R_L)=(1,1)$: $P_H^*=[C_L+(\pi_H-\pi_L)r]/\pi_H-\varepsilon$, $D_H^*=1$, $\Pi_H^*=-C_H+C_L+(\pi_H-\pi_L)r-\pi_H\varepsilon$, $P_L^*=C_L/\pi_L$, $D_L^*=0$, $\Pi_L^*=0$. Consumer's expected utilities are $EU^*=\pi_L r-C_L+\pi_H\varepsilon$.

(2) $(R_H, R_L)=(0,1)$: $P_H^*=C_L+(\pi_H-\pi_L)r-\varepsilon$, $D_H^*=1$, $\Pi_H^*=-C_H+C_L+(\pi_H-\pi_L)r-\varepsilon$, $P_L^*=C_L/\pi_L$, $D_L^*=0$, $\Pi_L^*=0$. Consumer's expected utilities are $EU^*=\pi_L r-C_L+\varepsilon$.

(3) $(R_H, R_L)=(1,0)$: $P_H^*=[C_L+(\pi_H-\pi_L)r]/\pi_H-\varepsilon$, $D_H^*=1$, $\Pi_H^*=-C_H+C_L+(\pi_H-\pi_L)r-\pi_H\varepsilon$, $P_L^*=C_L$, $D_L^*=0$, $\Pi_L^*=0$. Consumer's expected utilities are $EU^*=\pi_L r-C_L+\pi_H\varepsilon$.

For each firm, although the respective equilibrium prices may not be the same in all cases, it has the same demand and almost the same profit with a sufficiently small difference. Consequently, each firm is indifferent between offering a refund and not offering. Note that the consumer's expected utility is almost the same with a sufficiently

small difference in all cases. Moreover, the respective difference in prices for each firm is independent of the other firm's refund strategy, but depends on whether it offers a refund or not.

B.3. Proof of Proposition 3.2

Suppose (P_H^*, P_L^*) is a separating equilibrium in which $P_H^* \neq P_L^*$. Then the consumer's belief on the equilibrium path is as follows:

If he sees $(P_A, P_B) = (P_H^*, P_L^*)$, he believes that firm A produces high-quality goods with probability 1, and firm B produces low-quality goods with probability 1.

If one firm deviates from his equilibrium price, then the consumer forms beliefs off the equilibrium path as follows:¹

(1) If both prices are higher than C_H , he believes that firm A produces goods of high quality with probability h and of low quality with probability $(1-h)$; and that firm B produces goods of high quality with probability $(1-h)$ and of low quality with probability h . In particular, if $(P_A, P_B) = (P, P)$ for any $P \geq C_H$, the beliefs is of high quality with probability $1/2$ and of low quality with probability $1/2$. (2) If one price is higher than C_H and the other lower, he believes that the good with price lower than C_H is of low quality with probability 1.²

¹ This essay focuses on the reconstruction of play consistent with a minimal number of deviations from the putative equilibrium profile. See, for example, Bagwell and Ramey (1991) and Hertzendorf and Overgaard (1998a, 1998b) for a more formal discussion of consistent reconstruction of play.

² This is from Definition 3.1 (6).

Under the individual rationality (IR) condition and the incentive compatibility (IC) condition, each firm maximizes its own profit and the consumer maximizes his expected utility in equilibrium. By the virtue of Bertrand Competition, the demand distribution across firms in equilibrium, D_H^* and D_L^* , is among $\{1,0\}$, $\{0,1\}$, and $\{1/2, 1/2\}$. That is, I have the following cases:

(1) $\text{Max}\{0, \pi_H r - P_H^*\} < \pi_L r - P_L^*$, so that $D_H^*=0$ and $D_L^*=1$:

(1a) $\forall P_L^* \in [C_H, \pi_L r]$, firm H can deviate from P_H^* to P_L^* . In other words, firm H can pool with firm L. Then, $D_H=1/2$, $D_L=1/2$, and $\Pi_H \geq 0 = \Pi_H^*$. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

(1b) $\forall P_L^* \in [C_L, C_H]$, the consumer knows that the good with price P_L^* is of low quality with probability 1. Firm H can deviate from P_H^* to $\tilde{P}_H = P_L^* + (\pi_H - \pi_L)r - \varepsilon$, which is still higher than C_H . Thus, the consumer correctly identifies the two goods and chooses the high-quality good. In this case, $D_H=1$, $D_L=0$ and $\Pi_H > 0 = \Pi_H^*$. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.³

(1c) $\forall P_L^* > \pi_L r$, there is no region belongs to this category.

(2) $\pi_H r - P_H^* > \text{Max}\{0, \pi_L r - P_L^*\}$, so that $D_H^*=1$ and $D_L^*=0$:

(2a) $\forall P_H^* \in [C_H, (\pi_H + \pi_L)r/2]$, firm L can deviate from P_L^* to P_H^* . In other words, firm L can pool with firm H. Then $D_L=1/2$, $D_H=1/2$ and $\Pi_L > 0 = \Pi_L^*$. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

(2b) $\forall P_H^* \in ((\pi_H + \pi_L)r/2, \pi_H r]$, firm L has no incentive to pool with firm H because it still gets no demand by doing so. But, it can deviate downward to $\tilde{P}_L = P_H^* - (\pi_H - \pi_L)r - \varepsilon$.

³ From the assumption of $C_H < \pi_L r$, $\pi_H r - \tilde{P}_H > 0$. Thus, the demand facing firm H is 1.

By doing so, firm L always gets the entire market no matter what the consumer's beliefs off the equilibrium path h is. That is, firm L gets the entire market even in the worst case in which it is correctly identified as firm L. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

(2c) $\forall P_H^* > \pi_H r$, there is no region belongs to this category.

(3) $\pi_H r - P_H^* = \pi_L r - P_L^* > 0$, so that $D_H^* = 1/2$ and $D_L^* = 1/2$:

(3a) $\forall P_H^* \in [C_H, (\pi_H + \pi_L)r/2]$, since P_H^* is higher than P_L^* , firm L has an incentive to pool with firm H, in which it gets still half of the market but a higher profit. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

(3b) $\forall P_H^* > (\pi_H + \pi_L)r/2$, using the same argument as in (2b), firm L has an incentive to deviate a little bit downward to $\tilde{P}_L = P_H^* - (\pi_H - \pi_L)r - \varepsilon$. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

Summary: From the above analysis of (1), (2), and (3), there is no separating equilibrium in the no-refund subgame.

B.4. Proof of Proposition 3.3

Suppose that there exists a pooling equilibrium $(P_H, P_L) = (P^*, P^*)$. Since firm H will never set its price under C_H and the consumer will never buy a product if P^* is higher than his average valuation about these two products, the only possible region for a pooling equilibrium price is $[C_H, (\pi_H + \pi_L)r/2]$. Then the consumer's belief on the equilibrium path is

Each good is of high quality with probability 1/2 and of low quality with probability 1/2.

Let the consumer's out-of-equilibrium beliefs of seeing P^* and $\tilde{P}$ be as follows:

Whoever deviates to a higher $\tilde{P}$ than P^* is of high quality with probability $1/2$ and of low quality with probability $1/2$. Whoever deviates to a lower $\tilde{P}$ than P^* but higher than C_H is of high quality with probability m and of low quality with probability $(1-m)$. It is the low-quality firm who deviates to a price lower than C_H .

Since there are only two firms in the market, this belief implies that whoever stays at P^* is of high quality with probability $(1-m)$ and of low quality with probability m . Also note that m is a function of P^* .

Under this system of beliefs, no firm has an incentive to deviate upward. Hence, I only need to check the downward deviation.

(1) It is firm L who deviates downward to $\tilde{P}$, where $\tilde{P} < C_H$:

Then the consumer knows it is firm L who deviates and he assigns correct valuation to each good. Thus, the profits for firm L with deviating price is as follows:

$$\Pi_L(\text{deviating}) = \begin{cases} 1[(C_H - \varepsilon) - C_L] & \text{if } P^* - (\pi_H - \pi_L)r \geq C_H \\ 1\{[P^* - (\pi_H - \pi_L)r - \varepsilon] - C_L\} & \text{if } P^* - (\pi_H - \pi_L)r \in (C_L, C_H) \\ \frac{1}{2}[C_L - C_L] & \text{if } P^* - (\pi_H - \pi_L)r = C_L \\ 0[\text{any price} - C_L] & \text{if } P^* - (\pi_H - \pi_L)r < C_L \end{cases}$$

It can be checked for any equilibrium price that the deviating profit is lower than the equilibrium profit, which is $[P^* - C_L]/2$, for firm L. Therefore, firm L will never deviate to $\tilde{P} < C_H$ to reveal its type.

(2) $P^* = C_H$:

Since firm H cannot deviate downward, $m(C_H) = 0$. From (1), firm L will not deviate either. Thus, this price is supported as a pooling equilibrium.

(3) It is some firm which deviates downward to $\tilde{P}$, where $P^* > \tilde{P} \geq C_H$:

Then the consumer compares among $[m \pi_H + (1-m) \pi_L]r - \tilde{P}$, $[(1-m) \pi_H + m \pi_L]r - P^*$ and 0.

First, for the deviating firm to get the entire market, the following inequality must hold.

$$[m \pi_H + (1-m) \pi_L]r - \tilde{P} > \text{Max}\{0, [(1-m) \pi_H + m \pi_L]r - P^*\}$$

This implies that

$$\tilde{P} < [m \pi_H + (1-m) \pi_L]r \quad \text{if } 0 \geq [(1-m) \pi_H + m \pi_L]r - P^*,$$

$$\text{or } \tilde{P} < P^* + (2m-1)(\pi_H - \pi_L)r \quad \text{if } 0 < [(1-m) \pi_H + m \pi_L]r - P^*.$$

Second, to maximize its profit, the deviating firm will choose $\tilde{P}$ as high as possible.

Hence,

$$\tilde{P} = [m \pi_H + (1-m) \pi_L]r - \varepsilon \quad \text{if } 0 \geq [(1-m) \pi_H + m \pi_L]r - P^*,$$

$$\text{or, } \tilde{P} = P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon \quad \text{if } 0 < [(1-m) \pi_H + m \pi_L]r - P^*.$$

Finally, According to the concept of sequential equilibrium, to support this price as a pooling equilibrium, this deviating profit must be less than the equilibrium profit so that it has no incentive to deviate. Therefore, if it is firm H who deviates,

$$\{[m \pi_H + (1-m) \pi_L]r - \varepsilon - C_H\} \cdot 1 < \frac{1}{2}(P^* - C_H) \quad \text{if } 0 \geq [(1-m) \pi_H + m \pi_L]r - P^*, \quad (\text{B1})$$

$$\text{or, } \{P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon - C_H\} \cdot 1 < \frac{1}{2}(P^* - C_H) \quad \text{if } 0 < [(1-m) \pi_H + m \pi_L]r - P^*. \quad (\text{B2})$$

If it is firm L who deviates,

$$\{[m \pi_H + (1-m) \pi_L]r - \varepsilon - C_L\} \cdot 1 < \frac{1}{2}[P^* - C_L] \quad \text{if } 0 \geq [(1-m) \pi_H + m \pi_L]r - P^*, \quad (\text{B3})$$

$$\text{or, } \{P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon - C_L\} \cdot 1 < \frac{1}{2}[P^* - C_L] \quad \text{if } 0 < [(1-m) \pi_H + m \pi_L]r - P^*. \quad (\text{B4})$$

For a price to be a pooling equilibrium, neither firm can have an incentive to deviate. That is, either both the conditions of (B1) and (B3) must hold at the same time, or both the conditions of (B2) and (B4) must hold at the same time.

(3a) From (B1) and (B3),

$$0 \geq [(1-m) \pi_H + m \pi_L]r - P^*, \quad (\text{B5})$$

$$\{[m \pi_H + (1-m) \pi_L]r - \varepsilon - C_H\} \cdot 1 < \frac{1}{2} [P^* - C_H], \quad (\text{B6})$$

$$\{[m \pi_H + (1-m) \pi_L]r - \varepsilon - C_L\} \cdot 1 < \frac{1}{2} [P^* - C_L], \quad (\text{B7})$$

$$\tilde{P} < P^*, \text{ that is, } [m \pi_H + (1-m) \pi_L]r - \varepsilon < P^*. \quad (\text{B8})$$

First, from (B5), $m(\pi_H - \pi_L)r \geq \pi_H r - P^*$, where $P^* \in [C_H, (\pi_H + \pi_L)r/2]$. Since $m(P^*) \in [0, 1]$, when $P^* < \pi_L r$, the following condition must hold:

$$m(\pi_H - \pi_L)r \geq \pi_H r - P^* > (\pi_H - \pi_L)r.$$

This implies that $m(P^*) > 1$ when $P^* < \pi_L r$, which leads to a contradiction. Therefore, the region for (B5) to hold is

$$m(P^*) \in \left[\frac{\pi_H r - P^*}{(\pi_H - \pi_L)r}, 1 \right], \text{ where } P^* \in [\pi_L r, (\pi_H + \pi_L)r/2]. \quad (\text{B9})$$

Second, combine (B5) and (B8), $\pi_H r - P^* \leq m(\pi_H - \pi_L)r \leq P^* - \pi_L r$. It implies $(\pi_H + \pi_L)r/2 \leq P^*$. Thus, $P^* = (\pi_H + \pi_L)r/2$. Third, from (B6) and (B7), (B6) is satisfied if (B7) is satisfied. Hence, I only need to check (B7). From (B7), the following condition must hold:

$$m(P^*) < \frac{\frac{1}{2}(P^* + C_L) - \pi_L r + \varepsilon}{(\pi_H - \pi_L)r} \quad (\text{B10})$$

From (B9) and (B10), the following condition must hold for $m(P^*)$.

$$\frac{\frac{1}{2}(P^* + C_L) - \pi_L r}{(\pi_H - \pi_L)r} \geq \frac{\pi_H r - P^*}{(\pi_H - \pi_L)r}, \quad \text{where } P^* = (\pi_H + \pi_L)r/2. \quad (\text{B11})$$

The numerator of (B11) shows LHS < RHS, which implies (B11) does not hold.

Therefore, there is no $m(P^*)$ existing for the case of (B1) and (B3).

(3b) From (B2) and (B4), P^* and $\tilde{P}$ must satisfy the following conditions:

$$0 < [(1-m) \pi_H + m \pi_L]r - P^*, \quad (\text{B12})$$

$$\{P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon - C_H\} \cdot 1 < \frac{1}{2}(P^* - C_H), \quad (\text{B13})$$

$$\{P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon - C_L\} \cdot 1 < \frac{1}{2}[P^* - C_L], \quad (\text{B14})$$

$$\tilde{P} < P^*, \text{ that is, } P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon < P^*, \quad (\text{B15})$$

$$\tilde{P} \geq C_H, \text{ that is, } P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon \geq C_H. \quad (\text{B16})$$

From (B15), $(2m-1)(\pi_H - \pi_L)r \leq 0$, which implies $m \leq 1/2$. From (B12), $m(\pi_H - \pi_L)r < \pi_H r - P^*$,

or $m(P^*) < \frac{\pi_H r - P^*}{(\pi_H - \pi_L)r}$, where $P^* \in [C_H, (\pi_H + \pi_L)r/2]$. Since $\frac{\pi_H r - P^*}{(\pi_H - \pi_L)r} \geq 1/2 \forall P^* \leq$

$(\pi_H + \pi_L)r/2$, combined with (B15), $m(P^*) \leq 1/2 \forall P^* \in [C_H, (\pi_H + \pi_L)r/2]$.

From (B13) and (B14), (B13) is satisfied if (B14) is satisfied. Thus, I only need to check (B14). From (B14) and (B16), $P^* > 2C_H - C_L$ must hold. Thus, there exists no P^* for nontrivial $m(P^*)$ if $2C_H - C_L > (\pi_H + \pi_L)r/2$. Suppose $2C_H - C_L \leq (\pi_H + \pi_L)r/2$. From (B14), the following condition must hold.

$$m(P^*) < \frac{1}{2} \left[1 - \frac{1}{2} \frac{P^* - C_L}{(\pi_H - \pi_L)r} + \frac{\varepsilon}{(\pi_H - \pi_L)r} \right] \quad (\text{B17})$$

First, in order to have a solution for $m(P^*)$, the RHS of (B17) must be nonnegative,

which implies that $P^* < 2(\pi_H - \pi_L)r + C_L$ must hold. Note that $2C_H - C_L < 2(\pi_H - \pi_L)r + C_L$.

Hence, the region of P^* to satisfy this condition is $P^* \in [2C_H - C_L, \min\{2(\pi_H - \pi_L)r + C_L,$

$(\pi_H + \pi_L)r/2\}]$. Second, for the above region of P^* , $\frac{1}{2} \left[1 - \frac{1}{2} \frac{P^* - C_L}{(\pi_H - \pi_L)r} \right]$ is always less than

$1/2$. Therefore, the solution for $m(p^*)$ given that $2C_H - C_L \leq (\pi_H + \pi_L)r/2$ is

$$m(P^*) \in [0, \frac{1}{2} \left[1 - \frac{1}{2} \frac{P^* - C_L}{(\pi_H - \pi_L)r} \right]], \text{ for } P^* \in [2C_H - C_L, \min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\}]. \quad (\text{B18})$$

$\forall P^* \in (C_H, 2C_H - C_L]$, $m=0$ suffices to support them as pooling equilibria, because this belief requires the deviating price lower than C_H in order to seize the whole market demand, which has been shown no such incentive of deviation for each firm.

The relation between $m(P^*)$ and P^* given $2C_H - C_L \leq (\pi_H + \pi_L)r/2$ is depicted in Figure 3.3. It shows that there are multiple pooling equilibria in this subgame. Because firms are first movers, adopting the Pareto-Perfection argument, they will choose to pool at the highest possible price, which is $\min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\}$, to maximize their profits. The only out-of-equilibrium belief to support this price as a pooling equilibrium is $m=0$. Note that when $2C_H - C_L > (\pi_H + \pi_L)r/2$, the shaded triangle area in Figure 3.3 disappears and only a line segment $[C_H, (\pi_H + \pi_L)r/2]$ on $m=0$ is left in Figure 3.3. In this case, the pooling equilibrium surviving the Pareto-Perfection argument is $(\pi_H + \pi_L)r/2$. Therefore, the unique refined pooling equilibrium outcome surviving the Pareto-Perfection argument in this subgame is:

$\{P_H, P_L, \text{what the consumer buys}\} = \{\min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\}, \min\{2(\pi_H - \pi_L)r + C_L, (\pi_H + \pi_L)r/2\}, \text{half of good-A and half of good-B}\}$.

Summary: There exist multiple pooling equilibria when both firms play the no-refund strategies. Adopting the Pareto-Perfection argument, both firms will pool at $2(\pi_H - \pi_L)r + C_L$ or $(\pi_H + \pi_L)r/2$, whichever is smaller.

B.5. Proof of Lemma 3.2

When only one firm offers refunds, there is no pooling equilibrium since firms' refund policies are different. Suppose the separating equilibria are:

$$\{(P_{IH}, 1), (P_{OL}, 0)\} \text{ and } \{(P_{OH}, 0), (P_{IL}, 1)\},$$

where the first contract is firm H's contract and the second firm L's. '1' means that the firm offers refunds and '0' means not.

The complete information price sets, which are $\{((\pi_H - \pi_L)r + C_L - \varepsilon)/\pi_H, 1), (C_L, 0)\}$ and $\{((\pi_H - \pi_L)r + C_L - \varepsilon), (C_L/\pi_L, 1)\}$, cannot be sustained as the equilibria under incomplete information. Since $\frac{(\pi_H - \pi_L)r + C_L}{\pi_H} > \frac{C_L}{\pi_L} > \frac{C_H}{\pi_H}$ and $C_L < (\pi_H - \pi_L)r + C_L - \varepsilon$, although firm H has no incentive to mimic firm L, firm L always has an incentive to mimic firm H. For instance, firm L can deviate from P_{0L} to P_{0H} or from P_{1L} to P_{1H} so that the consumer will be confused with the deviating firm and the non-deviating firm. In other words, when the consumer observes $\{((\pi_H - \pi_L)r + C_L - \varepsilon)/\pi_H, 1), ((\pi_H - \pi_L)r + C_L - \varepsilon), 0\}$, he does not know who is mimicking although he knows that one of the two firms is mimicking. Then, the consumer will revert to his prior beliefs about each good as 1/2 probability of high quality and 1/2 probability of low quality. Under this belief, the consumer will buy 1/2 of good-A and 1/2 of good-B. Thus, firm L gets a higher profit by deviating. Hence, the complete information price pairs cannot be sustained under incomplete information.

B.6. Proof of Lemma 3.3

To avoid the mimicking by firm L, firm H has to distort its price so that firm L is unable to mimic. It is impossible to reach this goal by an upward distortion. So, the only way is to distort downwards. Since firm H has cost disadvantage in the no-refund strategy and cost advantage in the price-refund strategy, it can set $P_{1H} = C_L/\pi_L - \varepsilon$ and keep P_{0H} the same as in the complete information. This price P_{1H} strongly signals the consumer that the good with this price is of high quality. Thus, even if firm L sets its

price P_{0L} as P_{0H} , it cannot fool the consumer. On the other hand, firm L will not set the price P_{1L} as P_{1H} because it is lower than its cost. Finally, it can be checked that firm H has no incentive to deviate. Therefore, the separating equilibria in the incomplete information structure when only one firm offers refunds are:

$$\begin{aligned}\{(1, P_{1H}), (0, P_{0L})\} &= \{(1, C_L/\pi_L - \varepsilon), (0, C_L)\} \text{ and} \\ \{(0, P_{0H}), (1, P_{1L})\} &= \{(0, (\pi_H - \pi_L)r + C_L - \varepsilon), (1, C_L/\pi_L)\}.\end{aligned}$$

B.7. Proof of Lemma 3.4

This proof is similar to that of Proposition 3.2. This time, however, I show that there exists a unique separating equilibrium.

Suppose (P_H^*, P_L^*) is an separating equilibrium in which $P_H^* \neq P_L^*$, then the consumer's beliefs on the equilibrium path are as follows:

If he sees $(P_A, P_B) = (P_H^*, P_L^*)$, he believes that the firm A produces high-quality goods, and the firm B produces low-quality goods.

If one firm deviates from his equilibrium price, then the consumer forms beliefs off the equilibrium path as follows:

(1) If both prices are higher than C_L/π_L , he believes that the good A is of high quality with probability h and of low quality with probability $(1-h)$; the good B is of high quality with probability $(1-h)$ and of low quality with probability h . (2) If one price is higher than C_L/π_L and the other lower, he believes that the good with price lower than C_L/π_L is of high quality with probability 1.

Under the individual rationality (IR) condition and the incentive compatibility (IC) condition, each firm maximizes its own profits and the consumer maximizes his

expected utility in equilibrium. By the virtue of Bertrand Competition, the demand distribution across firms in equilibrium, D_H^* and D_L^* , is among $\{1,0\}$, $\{0,1\}$, and $\{1/2, 1/2\}$. That is, I have the following cases:

(1) $\text{Max}\{0, \pi_H r - \pi_H P_H^*\} < \pi_L r - \pi_L P_L^*$, so that $D_H^*=0$ and $D_L^*=1$:

$\forall P_L^* \in [C_L/\pi_L, r)$, firm H can deviate from P_H^* to P_L^* . In other words, firm H can pool with firm L. Then, $D_H=1/2$, $D_L=1/2$, and $\Pi_H > 0 = \Pi_L^*$. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

(2) $\pi_H r - \pi_H P_H^* > \text{Max}\{0, \pi_L r - \pi_L P_L^*\}$, so that $D_H^*=1$ and $D_L^*=0$:

(2a) $\forall P_H^* \in [C_L/\pi_L, r)$, firm L can deviate from P_L^* to P_H^* . In other words, firm L can pool with firm H. Then $D_L=1/2$, $D_H=1/2$ and $\Pi_L \geq 0 = \Pi_L^*$. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

(2b) $\forall P_H^* \in [C_H/\pi_H, C_L/\pi_L)$, firm L cannot pool with firm H because of cost disadvantage.

Then, the consumer chooses the good H for lower price and higher quality. Firm H maximizes its separating profits in this price region and charges $P_H^* = C_L/\pi_L - \varepsilon$. However, firm H may choose to pool with firm L if the profits derived from the pooling price are higher. In other words, the separating equilibrium is sustained only if the pooling is lower than $[2C_L/\pi_L - C_H/\pi_H]$. In this case, firm L gets zero demand. Therefore, this equilibrium price pair is $(P_H^*, P_L^*) = (C_L/\pi_L - \varepsilon, P^*)$, where $P^* < [2C_L/\pi_L - C_H/\pi_H]$. From the proof of Lemma 3.5, the maximum pooling price is θ and the condition for the existence of a separating equilibrium is reduced to $(2\pi_H - \pi_L)(C_L/\pi_L) - C_H > (\pi_H - \pi_L)r$.

(3) $\pi_H r - \pi_H P_H^* = \pi_L r - \pi_L P_L^* > 0$, so that $D_H^*=1/2$ and $D_L^*=1/2$:

$\forall P_L^* \in [C_L/\pi_L, r)$, $P_H^* = \frac{(\pi_H - \pi_L)r + \pi_L P_L^*}{\pi_H}$, which is higher than C_L/π_L and lower than r .

Since this P_H^* is higher than P_L^* , firm L has an incentive to pool with firm H, which gives firm L still half of the market demand but higher profits. Therefore, this price pair (P_H^*, P_L^*) is not an equilibrium pair.

Summary: From the above analysis of (1), (2), and (3), there exists a unique separating equilibrium in the price-refund subgame if $(2\pi_H - \pi_L)(C_L/\pi_L) - C_H > (\pi_H - \pi_L)r$.

B.8. Proof of Lemma 3.5

This proof is similar to that of Proposition 3.3. I show that there may exist multiple pooling sequential equilibria under certain conditions. In this case, I adopt the Perfect-Perfection argument to obtain the unique refined pooling equilibrium.

Suppose that there exists a pooling equilibrium $(P_H, P_L) = (P^*, P^*)$. Then it must be $P^* \in [C_L/\pi_L, r)$. Thus, the consumer's belief on the equilibrium path is:

Each good is of high quality with probability 1/2 and of low quality with probability 1/2.

Let the consumer's out-of-equilibrium belief of seeing P^* and $\tilde{P}$ be:

Whoever deviates to a higher $\tilde{P}$ than P^* is of high quality with probability 1/2 and of low quality with probability 1/2. Whoever deviates to a lower $\tilde{P}$ than P^* but higher than C_L/π_L is of high quality with probability m and of low quality with probability $(1-m)$. It is the high-quality firm who deviates to a price lower than C_L/π_L .

Since there are only two firms in the market, this belief implies that whoever stays at P^* is of high quality with probability $(1-m)$ and of low quality with probability m . Also note that m is a function of P^* .

Under this system of out-of-equilibrium beliefs, no firm has an incentive to deviate upwards. Thus, I only need to check the downward deviation.

(1) It is firm H who deviates downwards to $\tilde{P}$, with $\tilde{P} < C_L/\pi_L$:

Then the consumer knows that it is firm H who deviates and assigns correct valuation to each good. Thus, firm H's deviating profits are $\pi_H[(C_L/\pi_L - \epsilon) - C_H/\pi_H] \times 1 \forall P^* \leq r$. When $P^* \geq 2C_L/\pi_L - C_H/\pi_H$, the deviating profit is lower than the equilibrium profit, which is $\pi_H[P^* - C_H/\pi_H]/2$, for firm H. Thus, firm H will not deviate to reveal his type if the pooling price is high.

(2) $P^* = C_L/\pi_L$:

Since firm L cannot deviate downward, $m(C_L/\pi_L) = 1$. From case 1, firm H will deviate. Thus, this price cannot be sustained as a pooling equilibrium.

(3) It is some firm which deviates downwards to $\tilde{P}$, where $P^* > \tilde{P} \geq C_L/\pi_L$:

Then the consumer compares among $[m\pi_H + (1-m)\pi_L](r - \tilde{P})$, $[(1-m)\pi_H + m\pi_L](r - P^*)$ and 0. First, for the deviating firm to get the whole market, the following inequality must hold:

$$[m\pi_H + (1-m)\pi_L](r - \tilde{P}) > \text{Max}\{0, [(1-m)\pi_H + m\pi_L](r - P^*)\}.$$

This implies that

$$[m\pi_H + (1-m)\pi_L] \tilde{P} < [(1-m)\pi_H + m\pi_L]P^* + (2m-1)(\pi_H - \pi_L)r \quad \text{since } 0 \leq r - P^*.$$

Second, to maximize its profit, the deviating firm will choose $\tilde{P}$ as high as possible.

Hence,

$$\tilde{P} = \{[(1-m)\pi_H + m\pi_L]P^* + (2m-1)(\pi_H - \pi_L)r - \epsilon\} / [m\pi_H + (1-m)\pi_L].$$

Finally, according to the concept of sequential equilibrium, to support this price as a pooling equilibrium, this deviating profit must be less than the equilibrium profit so that it has no incentive to deviate. Therefore, to deter firm H from deviating,

$$\pi_H \left\{ \frac{(2m-1)(\pi_H - \pi_L)r + [(1-m)\pi_H + m\pi_L]P^*}{m\pi_H + (1-m)\pi_L} - \frac{C_H}{\pi_H} - \varepsilon \right\} \times 1 < \pi_H \left(P^* - \frac{C_H}{\pi_H} \right) \times \frac{1}{2} \quad (\text{B19})$$

Alternatively, to deter firm L from deviating,

$$\pi_L \left\{ \frac{(2m-1)(\pi_H - \pi_L)r + [(1-m)\pi_H + m\pi_L]P^*}{m\pi_H + (1-m)\pi_L} - \frac{C_L}{\pi_L} - \varepsilon \right\} \times 1 < \pi_L \left(P^* - \frac{C_L}{\pi_L} \right) \times \frac{1}{2} \quad (\text{B20})$$

For a price to be a pooling equilibrium, neither firm can have an incentive to deviate. That is, both of (B19) and (B20) must hold. Also, two more conditions are required as follows:

$$\tilde{P} < P^*, \text{ or } \{[(1-m)\pi_H + m\pi_L]P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon\} / [m\pi_H + (1-m)\pi_L] < P^*. \quad (\text{B21})$$

$$\tilde{P} \geq C_L/\pi_L, \text{ or } \{[(1-m)\pi_H + m\pi_L]P^* + (2m-1)(\pi_H - \pi_L)r - \varepsilon\} / [m\pi_H + (1-m)\pi_L] \geq C_L/\pi_L. \quad (\text{B22})$$

From (B21), $(2m-1)(\pi_H - \pi_L)(r - P^*) \leq 0$ must hold. It can hold either by $m=1/2$, $m < 1/2$, or $r \leq P^*$.

(3a) $r \leq P^*$: Combined with the condition of $r \geq P^*$, thus $P^* = r$. Since (B20) is satisfied if (B19) is satisfied, I only need to check (B19). After rearranging, (B19) becomes:

$$m(P^*) < \frac{2(\pi_H - \pi_L)r - (2\pi_H - \pi_L)P^* + \pi_L \frac{C_H}{\pi_H} + 2\pi_L \varepsilon}{(\pi_H - \pi_L)(4r - 3P^* - \frac{C_H}{\pi_H} - 2\varepsilon)} \equiv \bar{m}(P^*) \quad (\text{B23})$$

When $P^* = r$, the RHS of (B23) is negative, which leads to a contradiction. It implies that firm H has an incentive to deviate.

(3b) $m=1/2$: It contradicts to (B19).

(3c) $m < 1/2$: From (B23), which is the rearrangement of (B19), the denominator in the RHS is positive. Thus, the numerator in the RHS of (B23) needs to be nonnegative to have a solution for $m(P^*)$. Consequently, there is no such P^* if

$\theta \equiv \frac{2(\pi_H - \pi_L)r + \pi_L \frac{C_H}{\pi_H}}{2\pi_H - \pi_L} < \frac{C_L}{\pi_L}$. Suppose the opposite inequality. Note that $\bar{m}(C_L/\pi_L) < 1/2$,

$\bar{m}'(P^*) < 0$, $\bar{m}''(P^*) < 0$, and $\theta < r$. Thus, $m(P^*) \in [0, \bar{m}(P^*)]$ for $P^* \in [C_L/\pi_L, \theta]$.

Rearranging (B22), I obtain

$$m(P^*) > \frac{(\pi_H - \pi_L)r + C_L - HP^*}{(\pi_H - \pi_L)[2r - \frac{C_L}{\pi_L} - P^*]} \equiv \hat{m}(P^*).$$

It can be checked that $\hat{m}(C_L/\pi_L) > \bar{m}(C_L/\pi_L)$, $\hat{m}'(P^*) < 0$, $\hat{m}''(P^*) < 0$, and $\hat{m}^{-1}(0) < \bar{m}^{-1}(0)$.

Consequently, there are multiple pooling equilibria, as illustrated in Figure 3.4. Adopting the Pareto-Perfection argument, I obtain the unique refined pooling equilibrium with price at θ .

Nonetheless, this refined pooling equilibrium is immune only to those deviating prices higher than C_L/π_L . I still need to check if it is also immune to deviating prices lower than C_L/π_L . In other words, I need to check if firm H has an incentive to deviate to

$C_L/\pi_L - \varepsilon$. Comparing the deviating profits, $\pi_H \left(\frac{C_L}{\pi_L} - \frac{C_H}{\pi_H} - \varepsilon \right)$, with the putative

equilibrium profits, $\frac{1}{2} \pi_H \left(\theta - \frac{C_H}{\pi_H} \right)$, I obtain that firm H has no incentive to deviate if

$(2\pi_H - \pi_L)(C_L/\pi_L) - C_H < (\pi_H - \pi_L)r$, which is consistent with the result in (1). In this case,

$P^* = \theta$ is the unique pooling equilibrium surviving the Pareto-Perfection argument.

Summary: From the above analysis, there exists a unique refined pooling equilibrium in the price-refund subgame if $(2\pi_H - \pi_L)(C_L/\pi_L) - C_H < (\pi_H - \pi_L)r$ and the Pareto-Perfection argument is accepted.

B.9. Proof of Lemma 3.6

With complete information, social welfare $\equiv SW^* = \pi_H r - C_H$. With incomplete information, $SW = \pi_H r - C_H$ if $(\hat{R}_H, \hat{R}_L) = (1, 0)$; $SW = [(\pi_H + \pi_L)r - (C_H + C_L)]/2$ if $(\hat{R}_H, \hat{R}_L) = (0, 0)$ and $(1, 1)$. Note that $\pi_H r - C_H > [(\pi_H + \pi_L)r - (C_H + C_L)]/2$.

B.10. Proof of Proposition 3.8

Maximizing the profits of firm H with respect to P_H and R_H yields the best response of firm H: $\Gamma_H = (P_H, R_H)$ such that $2[1 - (1 - \pi_H)R_H]P_H = (\pi_H - \pi_L) + [1 - (1 - \pi_L)R_L]P_L$. Similarly, maximizing the profits of firm L with respect to P_L and R_L yields the best response of firm L: $\Gamma_L = (P_L, R_L)$ such that $[1 - (1 - \pi_H)R_H]P_H = (2\pi_H/\pi_L)[1 - (1 - \pi_L)R_L]P_L$. Solving for the fix-point of the best responses gives the complete information Nash equilibria.

B.11. Proof of Lemma 3.8

From condition (3) of Lemma 3.7, the profits of firm L given the strategy of firm H is $\Pi_L(\Gamma_L, \hat{\Gamma}_H, \mu(\Gamma_L, \hat{\Gamma}_H), \pi_H, \pi_L) = [1 - (1 - \pi_L)R_L]P_L D_L$, where

$$D_L = \frac{[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H - [1 - (1 - \pi_L)R_L]P_L}{\pi_H - \pi_L} - \frac{[1 - (1 - \pi_L)R_L]P_L}{\pi_L}.$$

Maximizing Π_L with respect to P_L and R_L yields the result.

B.12. Proof of Proposition 3.9

(1) $R_H^* = 0$: then $P_H^* = 2\pi_H(\pi_H - \pi_L)/(4\pi_H - \pi_L)$. Let $\Pi_L^0 \equiv \Pi_L((P_H^*, 0), (P_H^*, 0), 1/2, \pi_H, \pi_L) =$

$$(1/2)P_H^* [1 - (2P_H^*/(\pi_H + \pi_L))] = \frac{\pi_H(\pi_H - \pi_L)}{4\pi_H - \pi_L} \left(1 - \frac{4\pi_H(\pi_H - \pi_L)}{(\pi_H + \pi_L)(4\pi_H - \pi_L)} \right). \text{ Comparing}$$

this expression with $\Pi_L^* = \pi_H \pi_L (\pi_H - \pi_L) / (4\pi_H - \pi_L)^2$ shows that firm L will deviate if

$$\Pi_L^0 \geq \Pi_L^*, \text{ or if } 1 - \frac{4\pi_H (\pi_H - \pi_L)}{(\pi_H + \pi_L)(4\pi_H - \pi_L)} \geq \frac{\pi_L}{4\pi_H - \pi_L}. \text{ It reduces to } 2\pi_L(3\pi_H - \pi_L) \geq 0,$$

which is the case for all $\pi_L \in (0, \pi_H)$.

(2) $R_H^* = 1$: then $P_H^* = 2(\pi_H - \pi_L) / (4\pi_H - \pi_L)$. Let $\Pi_L^0 = \Pi_L((P_H^*, 1), (P_H^*, 1), 1/2, \pi_H, \pi_L) =$

$$(1/2)P_H^* [1 - (2P_H^* / (\pi_H + \pi_L))R_H^*] = \pi_L \frac{\pi_H - \pi_L}{4\pi_H - \pi_L} \left(1 - \frac{2(\pi_H - \pi_L)}{4\pi_H - \pi_L} \right). \text{ Comparing this}$$

expression with $\Pi_L^* = \pi_H \pi_L (\pi_H - \pi_L) / (4\pi_H - \pi_L)^2$ shows that firm L will deviate if $\Pi_L^0 \geq \Pi_L^*$,

which is $\frac{2\pi_H + \pi_L}{4\pi_H - \pi_L} \geq \frac{\pi_H}{4\pi_H - \pi_L}$. This is the case for all $\pi_L \in (0, \pi_H)$.

B.13. Proof of Lemma 3.10

Numerical analysis reveals that: (1) $\bar{P}_H > \bar{\bar{P}}_H$ for $\pi_L > 0.1561$ if $\hat{R}_L = 0$ and $\hat{R}_H = 0$;

(2) $\bar{P}_H > \bar{\bar{P}}_H$ for $\pi_L \geq 0$ if $\hat{R}_L = 0$ and $\hat{R}_H = 1$; (3) $\bar{P}_H > \bar{\bar{P}}_H$ for $\pi_L > 0.45$ if

$\hat{R}_L = 1$ and $\hat{R}_H = 0$; and (4) $\bar{P}_H > \bar{\bar{P}}_H$ for $\pi_L > 0.6$ if $\hat{R}_L = 1$ and $\hat{R}_H = 1$. Consequently,

$\bar{P}_H(\cdot) > \bar{\bar{P}}_H(\cdot, 0)$ for any $\hat{R}_H$ if $\pi_L > 0.1561$, and $\bar{P}_H(\cdot) > \bar{\bar{P}}_H(\cdot, 1)$ for any $\hat{R}_H$ if $\pi_L > 0.6$.

B.14. Proof of Theorem 3.1

Define out-of-equilibrium beliefs such that $\mu((\hat{P}_H, \hat{R}_H), (P, R)) = 1$ for all $(P, R) \notin$

$\{(\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L)\}$ and such that $\mu((P, R), (\hat{P}_L, \hat{R}_L)) = 0$ for all $(P, R) \notin \{(\hat{P}_H, \hat{R}_H),$

$(\hat{P}_L, \hat{R}_L)\}$. These are the beliefs that can support the largest possible set of equilibria.

Intuitively, it requires consumers to view any deviating firm as firm L. Given the above

out-of-equilibrium beliefs, I need to check that neither firm has an incentive to deviate

from its respective strategy.

(1) Firm L has no incentive to deviate:

Any deviation to $(P, R) \neq (\hat{P}_H, \hat{R}_H)$ leaves beliefs unaffected. Since by construction firm L is already maximizing its profits subject to these beliefs, such a deviation must be suboptimal. On the other hand, a deviation to $(P, R) = (\hat{P}_H, \hat{R}_H)$ implies that $\mu((\hat{P}_H, \hat{R}_H), (\hat{P}_H, \hat{R}_H)) = 1/2$ by the definition of a sequential equilibrium. However, since by construction $(\hat{P}_H, \hat{R}_H) \in B^S(\pi_L)$, such a deviation is also suboptimal.

(2) Firm H has no incentive to deviate:

(2a) Firm H may deviate to $(P, R) = (\hat{P}_L, \hat{R}_L)$, which implies that $\mu((\hat{P}_L, \hat{R}_L), (\hat{P}_L, \hat{R}_L)) = 1/2$ by the definition of a sequential equilibrium. However, since by construction $(\hat{P}_H, \hat{R}_H) \in H^S(\pi_L)$, such a deviation is suboptimal.

(2b) Firm H may deviate to (P, R) such that $[1 - (1 - \pi_L)R]P \geq [1 - (1 - \pi_H)\hat{R}_L]\hat{P}_L$. However, firm H will get no demand by being identified as the low-quality firm and charging higher $[1 - (1 - Q(\mu))R_q]P_q$ than firm L. Therefore, such a deviation is suboptimal, compared to the positive profits in the putative separating equilibrium.

(2c) Firm H may deviate to (P, R) such that $[1 - (1 - \pi_L)R]P < [1 - (1 - \pi_H)\hat{R}_L]\hat{P}_L$. Then, the demand faced firm H when believed to offer low quality is

$$D_H(\Gamma, \hat{\Gamma}_L, 0, \pi_H, \pi_L) = \frac{[1 - (1 - \pi_H)\hat{R}_L]\hat{P}_L - [1 - (1 - \pi_L)R]P}{\pi_H - \pi_L} - \frac{[1 - (1 - \pi_L)R]P}{\pi_L},$$

and the payoff is $[1 - (1 - \pi_H)R]PD_H(\Gamma, \hat{\Gamma}_L, 0, \pi_H, \pi_L)$.

Maximizing the payoff with respect to P given $R=0$ yields $P = (\pi_L / (2\pi_H)) [1 - (1 -$

$\pi_H \hat{R}_L \hat{P}_L$, which results in a payoff of $\frac{\pi_L [1 - (1 - \pi_H) \hat{R}_L]^2 (\hat{P}_L)^2}{4\pi_H (\pi_H - \pi_L)}$. Similarly,

maximizing the payoff with respect to P given $R=1$ yields $P=(1/(2\pi_H))[1-(1-\pi_H)\hat{R}_L]\hat{P}_L$,

which results in a payoff of $\frac{[1 - (1 - \pi_H) \hat{R}_L]^2 (\hat{P}_L)^2}{4(\pi_H - \pi_L)}$. Comparing these two payoffs, I

obtain the optimal strategy of firm H in this case is $(P, R)=((1/(2\pi_H))[1-(1-\pi_H)\hat{R}_L]\hat{P}_L, 1)$.

But, recall that $[1-(1-\pi_L)\hat{R}_L]\hat{P}_L = (\pi_L/(2\pi_H))[1-(1-\pi_H)\hat{R}_H]\hat{P}_H$, which implies that firm H

will best deviate from $(\hat{P}_H, \hat{R}_H)$ to $((\pi_L/(4\pi_H^2))[1-(1-\pi_H)\hat{R}_H]\hat{P}_H, 1)$ if $\hat{R}_L=0$, and from

$(\hat{P}_H, \hat{R}_H)$ to $((\pi_H/(4\pi_H^2))[1-(1-\pi_H)\hat{R}_H]\hat{P}_H, 1)$ if $\hat{R}_L=1$. Thus, the maximum deviation

payoff of firm H is $\frac{\pi_L^2 [1 - (1 - \pi_H) \hat{R}_H]^2 (\hat{P}_H)^2}{16\pi_H^2 (\pi_H - \pi_L)}$ if $\hat{R}_L=0$, and $\frac{[1 - (1 - \pi_H) \hat{R}_H]^2 (\hat{P}_H)^2}{16(\pi_H - \pi_L)}$ if

$\hat{R}_L=1$.

I wish to show that the maximum deviation payoff is lower than the putative equilibrium payoff for firm H so that it has no incentive to deviate. Hence, I am done if I can show that $\Pi_H((\hat{P}_H, \hat{R}_H), (\hat{P}_L, \hat{R}_L), 1, \pi_H, \pi_L) >$ the maximum deviation payoff. That is, I am trying to show that

$$\frac{[1 - (1 - \pi_H) \hat{R}_H]^2 (\hat{P}_H)^2}{2(\pi_H - \pi_L)} > \frac{\pi_L^2 [1 - (1 - \pi_H) \hat{R}_H]^2 (\hat{P}_H)^2}{8\pi_H^2 2(\pi_H - \pi_L)} \text{ for } \hat{R}_L = 0;$$

$$\text{and } \frac{[1 - (1 - \pi_H) \hat{R}_H]^2 (\hat{P}_H)^2}{2(\pi_H - \pi_L)} > \frac{1 [1 - (1 - \pi_H) \hat{R}_H]^2 (\hat{P}_H)^2}{8 2(\pi_H - \pi_L)} \text{ for } \hat{R}_L = 1.$$

Clearly, both of the above two expressions are true for those small π_L such that

$B^S(\pi_L) \cap T^S(\pi_L, \hat{R}_L)$ is nonempty.

B.15. Proof of Theorem 3.2

As in the proof of Theorem 3.1, I Define out-of-equilibrium beliefs such that $\mu((P, R), (\hat{P}, \hat{R}))=0$ for all $(P, R) \neq (\hat{P}, \hat{R})$. These are the beliefs that can support the largest possible set of equilibria. Intuitively, it requires consumers to view any deviating firm as firm L. Given the above out-of-equilibrium beliefs, I need to check that neither firm has an incentive to deviate from its respective strategy.

(1) Firm L has no incentive to deviate:

Any deviation to $(P, R) \neq (\hat{P}, \hat{R})$ corrects consumer beliefs. To deter firm L from deviating, it must be that $\Pi_L((\hat{P}, \hat{R}), (\hat{P}, \hat{R}), 1/2, \pi_H, \pi_L) > \Pi_L((P, R), (\hat{P}, \hat{R}), 0, \pi_H, \pi_L)$, which is equivalent to requiring that $(\hat{P}, \hat{R}) \in B(\pi_L)$. Note that $\pi_L[1 - (1 - \pi_H)\hat{R}] \leq \pi_H[1 - (1 - \pi_L)\hat{R}]$, which implies that $B(\pi_L) \supseteq B^P(\pi_L)$. Consequently, any $(\hat{P}, \hat{R}) \in B^P(\pi_L)$ also belongs to the set $B(\pi_L)$.

(2) Firm H has no incentive to deviate:

(2a) Firm H may deviate to (P, R) such that $[1 - (1 - \pi_L)R]P \geq [1 - (1 - \pi_H)\hat{R}]\hat{P}$. However, firm H will get no demand by being identified as the low-quality firm and charging higher $[1 - (1 - Q(\mu))R_q]P_q$ than firm L. Therefore, such a deviation is suboptimal, compared to the positive profits in the putative pooling equilibrium.

(2b) Firm H may deviate to (P, R) such that $[1 - (1 - \pi_L)R]P < [1 - (1 - \pi_H)\hat{R}]\hat{P}$. Then, following the same procedure as in the proof of Theorem 3.1, (2c), with the substitution of $(\hat{P}, \hat{R})$ to $(\hat{P}_L, \hat{R}_L)$, I obtain $(P, R) = ((1/(2\pi_H))[1 - (1 - \pi_H)\hat{R}]\hat{P}, 1)$ and the maximum

deviation profits are $\frac{[1 - (1 - \pi_H)\hat{R}]^2 (\hat{P})^2}{4(\pi_H - \pi_L)}$.

In order to deter firm H from deviating, it must be that $\Pi_H((\hat{P}, \hat{R}), (\hat{P}, \hat{R}), 1/2, \pi_H, \pi_L) > \Pi_H((P, R), (\hat{P}, \hat{R}), 0, \pi_H, \pi_L)$, where $(P, R) = ((1/(2\pi_H))[1 - (1 - \pi_H)\hat{R}]\hat{P}, 1)$. This can be

simplified as $1 - \frac{2\hat{P}}{\pi_H + \pi_L} \left(1 - (1 - \frac{\pi_H + \pi_L}{2})\hat{R} \right) > \frac{[1 - (1 - \pi_H)\hat{R}]\hat{P}}{2(\pi_H - \pi_L)}$, or

$$\hat{P} < \frac{1}{\frac{[1 - (1 - \pi_H)\hat{R}]}{2(\pi_H - \pi_L)} + \frac{2[1 - (1 - \frac{\pi_H + \pi_L}{2})\hat{R}]}{\pi_H + \pi_L}}, \text{ which is equivalent to requiring that } (\hat{P}, \hat{R}) \in$$

$B^P(\pi_L)$.

B.16. Proof of Lemma 3.11

(1) $\hat{R}_L = 0$ and $\hat{R}_H = 0$: then $\hat{P}_L = \frac{\pi_L}{2\pi_H}\hat{P}_H$. Numerical Analysis reveals that

$$\text{Max } \hat{P}_L = \frac{\pi_L}{2\pi_H} \bar{P}_H(0, 0; \pi_L) = \frac{\pi_L}{2\pi_H} \frac{4\pi_H^2(\pi_H^2 - \pi_L^2) - 1}{2\pi_H(2\pi_H - \pi_L)(\pi_H + \pi_L) - L^2(\pi_H - \pi_L)}$$

$$< \min \hat{P}_H = \bar{P}_H(0; \pi_L) = \frac{2\pi_H(\pi_H - \pi_L)(\pi_H + \pi_L)}{\pi_L(\pi_H + \pi_L) + 4\pi_H(\pi_H - \pi_L)}.$$

(2) $\hat{R}_L = 1$ and $\hat{R}_H = 1$: then $\hat{P}_L = \frac{1}{2}\hat{P}_H$. It is easy to check that

$$\text{Max } \hat{P}_L = \frac{1}{2}\bar{P}_H(1, 1; \pi_L) = \frac{3(\pi_H - \pi_L)}{7\pi_H - 3\pi_L} < \min \hat{P}_H = \bar{P}_H(1; \pi_L) = \frac{2(\pi_H - \pi_L)}{3\pi_H - 2\pi_L}.$$

From (1) and (2), the sets of possible separating equilibrium strategies for the two firms are disjoint.

B.17. Proof of Lemma 3.12

$$\text{With } \hat{R}_L = 0, \hat{P}_L = \frac{\pi_L}{2\pi_H}[1 - (1 - \pi_H)\hat{R}_H]\hat{P}_H.$$

(1) $\hat{R}_H=0$: then $\hat{P}_L = \frac{\pi_L}{2\pi_H} \hat{P}_H$, where $\hat{P}_H \in [\bar{P}_H(0; \pi_L), \bar{\bar{P}}_H(0; 0; \pi_L)]$. The profits of firm

H is $\Pi_H((P, 0), (\hat{P}_L, 0), 1, \pi_H, \pi_L) = P(1 - ((P - \hat{P}_L)/(\pi_H - \pi_L)))$. Maximizing with respect to P

and substituting in for $P = \hat{P}_H$ and $\hat{P}_L = \frac{\pi_L}{2\pi_H} \hat{P}_H$ yields $d\Pi_H/dP = 1 - ((4\pi_H - \pi_L)/(2\pi_H(\pi_H -$

$\pi_L))) \hat{P}_H < 0$, since $\hat{P}_H > P_H^*$. Thus, firm H will choose $\hat{P}_H = \bar{P}_H(0; \pi_L)$.

(2) $\hat{R}_H=1$: then $\hat{P}_L = \frac{\pi_L}{2} \hat{P}_H$, where $\hat{P}_H \in [\bar{P}_H(1; \pi_L), \bar{\bar{P}}_H(1; 0; \pi_L)]$. The profits of firm H is

$\Pi_H((P, 1), (\hat{P}_L, 0), 1, \pi_H, \pi_L) = \pi_H P(1 - ((\pi_H P - \hat{P}_L)/(\pi_H - \pi_L)))$. Maximizing with respect to P

and substituting in for $P = \hat{P}_H$ and $\hat{P}_L = \frac{\pi_L}{2} \hat{P}_H$ yields $d\Pi_H/dP = \pi_H[1 - ((4\pi_H - \pi_L)/(2(\pi_H -$

$\pi_L))) \hat{P}_H] < 0$, since $\hat{P}_H > P_H^*$. Thus, firm H will choose $\hat{P}_H = \bar{P}_H(1; \pi_L)$.

B.18. Proof of Lemma 3.13

It is similar to the proof of Lemma 3.12. With $\hat{R}_L=1$,

$$\hat{P}_L = \frac{1}{2\pi_H} [1 - (1 - \pi_H) \hat{R}_H] \hat{P}_H.$$

(1) $\hat{R}_H=0$: then $\hat{P}_L = \frac{1}{2\pi_H} \hat{P}_H$, where $\hat{P}_H \in [\bar{P}_H(0; \pi_L), \bar{\bar{P}}_H(0; 1; \pi_L)]$. The profits of firm H

is $\Pi_H((P, 0), (\hat{P}_L, 1), 1, \pi_H, \pi_L) = P(1 - ((P - \pi_L \hat{P}_L)/(\pi_H - \pi_L)))$. Maximizing with respect to P

and substituting in for $P = \hat{P}_H$ and $\hat{P}_L = \frac{1}{2\pi_H} \hat{P}_H$ yields $d\Pi_H/dP = 1 - ((4\pi_H - \pi_L)/(2\pi_H(\pi_H -$

$\pi_L))) \hat{P}_H < 0$, since $\hat{P}_H > P_H^*$. Thus, firm H will choose $\hat{P}_H = \bar{P}_H(0; \pi_L)$.

(2) $\hat{R}_H=1$: then $\hat{P}_L = \frac{1}{2} \hat{P}_H$, where $\hat{P}_H \in [\bar{P}_H(1; \pi_L), \bar{\bar{P}}_H(1; 1; \pi_L)]$. The profits of firm H is

$\Pi_H((P, 1), (\hat{P}_L, 1), 1, \pi_H, \pi_L) = \pi_H P (1 - ((\pi_H P - \pi_L \hat{P}_L) / (\pi_H - \pi_L)))$. Maximizing with respect to P and substituting in for $P = \hat{P}_H$ and $\hat{P}_L = \frac{1}{2} \hat{P}_H$ yields $d\Pi_H/dP = \pi_H [1 - ((4\pi_H - \pi_L) / (2(\pi_H - \pi_L))) \hat{P}_H] < 0$, since $\hat{P}_H > P_H^*$. Thus, firm H will choose $\hat{P}_H = \bar{P}_H(1; \pi_L)$.

B.19. Proof of Theorem 3.4

From Lemma 3.12, where $\hat{R}_L = 0$, the two strategies of firm H left are $(\bar{P}_H(0; \pi_L), 0)$ and $(\bar{P}_H(1; \pi_L), 1)$. Comparing the two profits of firm H associated with these two strategies reveals that the optimal strategy for firm H is $(\bar{P}_H(0; \pi_L), 0)$, where $\pi_L < 0.1561$.

B.20. Proof of Theorem 3.5

From Lemma 3.13, where $\hat{R}_L = 1$, the two strategies of firm H left are $(\bar{P}_H(0; \pi_L), 0)$ and $(\bar{P}_H(1; \pi_L), 1)$. Comparing the two profits of firm H associated with these two strategies reveals that the optimal strategy for firm H is $(\bar{P}_H(0; \pi_L), 0)$ for $\pi_L \leq 0.30$ and $(\bar{P}_H(1; \pi_L), 1)$ for $\pi_L \in [0.30, 0.60]$.

B.21. Proof of Theorem 3.6

Suppose a firm deviates from the putative pooling equilibrium $(\hat{P}, \hat{R})$ to another strategy $(\tilde{P}, \tilde{R})$. Consider the two cases associated with the values of $\hat{R}$.

(1) $\hat{R} = 0$: (1a) if the firm deviates to $\tilde{R} = 0$, consumer beliefs are the same, i.e., $1/2$. Then, consumers make their purchase decision by comparing $(\frac{\pi_H + \pi_L}{2})r - \tilde{P}$ with

$(\frac{\pi_H + \pi_L}{2})r - \hat{P}$. Clearly, each firm will undercut the price of its rival until matching its

marginal cost in order to seize the whole demand. Consequently, the only pooling equilibrium is (0, 0). (1b) if the firm deviates to $\tilde{R}=1$, consumer beliefs are 1. Then, in order to deter each firm from deviating, it must be the profits derived from the putative strategy no less than the maximum profits derived from the deviating strategy for each firm. It can be checked that there exists no $\hat{P}$ to satisfy such condition.

From (1a) and (1b), for $\hat{R}=0$, the deviating firm will choose to deviate to $\tilde{R}=1$ and earn higher profits. That is, the deviating firm will definitely deviate. Hence, this case is ruled out.

(2) $\hat{R}=1$: (2a) if the firm deviates to $\tilde{R}=0$, consumer beliefs are the same, i.e., 1/2. Then, consumers make their purchase decision by comparing $(\frac{\pi_H + \pi_L}{2})r - \tilde{P}$ with

$(\frac{\pi_H + \pi_L}{2})(r - \hat{P})$. Clearly, each firm will undercut the price of its rival until matching

its marginal cost in order to seize the whole demand. Consequently, the only pooling equilibrium is (0, 1). (2b) if the firm deviates to $\tilde{R}=1$, consumer beliefs are 1. Then, in order to deter each firm from deviating, it must be the profits derived from the putative strategy no less than the maximum profits derived from the deviating strategy for each firm. It can be checked that there exists no $\hat{P}$ to satisfy such condition.

From (2a) and (2b), for $\hat{R}=1$, the firm will choose to deviate to $\tilde{R}=1$ and earn higher profits. That is, the firm will definitely deviate. Hence, this case also is ruled out.

Since both cases are ruled out to support a pooling equilibrium, there exists no pooling equilibrium given the incentive argument.

B.22. Proof of Corollary 3.1

Recall that $\hat{P}_{H0} \in [\bar{P}_H(0; \pi_L), \bar{\bar{P}}_H(0; 0; \pi_L)]$. Thus, for $\pi_L \in (0, 0.1561)$ such that the set is nonempty, $\bar{P}_H(0; \pi_L) \leq \hat{P}_{H0}$. It is easy to check that $P_{H0}^* < \bar{P}_H(0; \pi_L)$, hence $P_{H0}^* < \hat{P}_{H0}$. Since $P_{L0}^* = \frac{\pi_L}{2\pi_H} P_{H0}^*$ and $\hat{P}_{L0} = \frac{\pi_L}{2\pi_H} \hat{P}_{H0}$, the upward distortion of $\hat{P}_{H0}$ implies the upward distortion of $\hat{P}_{L0}$ as well.

B.23. Proof of Corollary 3.2

For the comparison of $\hat{P}_H$ with P_H^* , note that in the case of $\hat{R}_L = 0$, I compare $\hat{P}_H = \bar{P}_H(0; \pi_L)$ to P_{H0}^* , since $\hat{R}_H = 0$; in the case of $\hat{R}_L = 1$ and $\pi_L \in (0, 0.3)$, I compare $\hat{P}_H = \bar{P}_H(0; \pi_L)$ to P_{H0}^* , since $\hat{R}_H = 0$; in the case of $\hat{R}_L = 1$ and $\pi_L \in (0.3, 0.6)$, I compare $\hat{P}_H = \bar{P}_H(1; \pi_L)$ to P_{H1}^* , since $\hat{R}_H = 1$. Similarly, for the comparison of $\hat{P}_L$ with P_L^* , note that in the case of $\hat{R}_L = 0$, I compare $\hat{P}_L = (\pi_L / (2\pi_H)) \bar{P}_H(0; \pi_L)$ to P_{L0}^* ; in the case of $\hat{R}_L = 1$ and $\pi_L \in (0, 0.3)$, I compare $\hat{P}_L = (1 / (2\pi_H)) \bar{P}_H(0; \pi_L)$ to P_{L1}^* ; in the case of $\hat{R}_L = 1$ and $\pi_L \in (0.3, 0.6)$, I compare $\hat{P}_L = (1/2) \bar{P}_H(1; \pi_L)$ to P_{L1}^* . It is easy to check all of the above comparisons that $\hat{P}_q > P_q^*$, $q \in \{H, L\}$.

B.24. Proof of Corollary 3.3

According to the proof of Corollary 3.2, the calculation is straightforward.

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BIOGRAPHICAL SKETCH

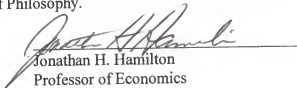
Hsiu-Chuan Jenny Yeh was born in January, 1968, in Taiwan. She received a bachelor's degree in mathematics in 1990 from National Taiwan University, and a Master of Business Administration degree in finance in 1993 from Fu Jen University. She began the doctoral program at the Economics Department of the University of Florida in fall 1995. She got married at the end of 1993 and gave birth to a lovely daughter in March, 2000.

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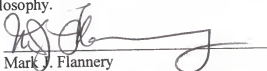
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